

## The Mathematics of Conflicts

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Strategic Games

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### Unit Description

This course is about the mathematics of conflicts and is generally associated with game theory. However, the author has shown that the mathematics of conflicts is not only applied to game theory analysis; as was the case when the author was involved in an actual legal dispute and applied the well-known Kelly Criterion when multiple outcomes exist (as typically used in casino games) to lawsuits to determine whether it is beneficial to file a lawsuit given there are risks involved if unsuccessful in court. The Kelly Criterion can also be used to determine whether it is worth having legal representation given the additional costs (although the chances of winning the lawsuit may be greater), a suitable amount for an out-of-court settlement through negotiation and a fair arbitration value. This is covered in chapter 2 of this book. Chapter 1 covers information on risk theory by providing the analysis of casino games, the Kelly Criterion when multiples outcomes exists and an application to video poker. Chapter 3 covers warfare conflicts by applying the well-defined hierarchical scoring structure in tennis to the hierarchical structure in warfare to optimize resources. Chapter 4 covers the underlying theory for two-person zero-sum games and provides applications to hierarchical games, tennis and poker. Chapter 5 covers information for two-person zero-sum games when there is risk involved. Risk-averse strategies are devised as well as the Kelly Equilibrium, which can be used as an alternative to the Nash Equilibrium. Chapter 6 covers information for two-person nonzero-sum games. The Nash Equilibrium, maximin strategies and the Pareto principle are devised. Applications are obtained for the Prisoner's Dilemma, strategic moves and evolutionary stable strategies. The Nash Arbitration Scheme is also devised with an application to the duopoly problem.

## Unit Outcomes

- Learn concepts in casino game analysis, Kelly Criterion, risk theory, recursion formulas, probability theory, distribution theory, utility theory, game theory, dispute resolution
- Become familiarized with Excel software by hands-on experience in building your own tennis calculator, which in turn assists in the understanding of probability and statistical concepts
- Apply operation research and game theory techniques to addressing real-world problems in employment disputes amongst many other areas.
- Have a greater appreciation of the legal system for resolving disputes
- Have a greater appreciation of casino games through the well-defined mathematical formulations
- Prepare students with quantitative skills for careers in law and defence

## Structure of the course

The course is divided into six parts. Part 1 defines risk theory where the analysis of casino games is derived, the Kelly Criterion with multiple outcomes to maximize the long-term growth of the bank, and an application to video poker. Part 2 defines dispute resolution in lawsuits where the mathematics of risk theory is used to determine a fair arbitration in civil law disputes. Part 3 defines optimizing strategies in warfare by applying the well-defined hierarchical scoring structure of tennis to the hierarchical structure of warfare. Part 4 defines two-person zero-sum games with applications to hierarchical games, tennis and poker. Part 5 defines two-person zero-sum games when there is risk involved. Risk-averse strategies are devised as well as the Kelly Equilibrium which can be used as an alternative to the Nash Equilibrium. Part 6 defines two-person nonzero-sum games. Prisoner's Dilemma, Evolutionary stable strategies, Nash Arbitration Scheme and the Duopoly problem are covered.

## Resources

### The textbooks for the course:

Barnett T (2019). The Mathematics of Conflicts. Strategic Games.

1. <http://strategicgames.com.au/conflicts.pdf>

Straffin P (1993). Game Theory and Strategy. The Mathematical Association of America.

### Calculators for the course:

Java tennis calculator

1. <http://strategicgames.com.au/TennisCalc.jar>

Excel tennis calculator using simulation for the distribution of points played in a set/match

2. <http://strategicgames.com.au/tennisdeucesim.xlsx>

Warfare calculator

3. <http://strategicgames.com.au/warfare.xlsx>

Game theory calculator

4. <http://strategicgames.com.au/conflicts.xlsx>

Lawsuits calculator

5. <http://strategicgames.com.au/lawsuits.xlsx>

Data for the course:

WinPoker

6. <http://www.zamzone.com/>

Gambit

7. <http://www.gambit-project.org/>

## Unit Schedule

Week	Part 1: Risk theory	Book	Chapter/Part	Calculator/Data
1	Casino games, Kelly Criterion, Video poker	1.	1.	6.
	<b>Part 2: Lawsuits</b>			
2	Dispute resolution, Litigation, Negotiation, Arbitration	1.	2.	5.
	<b>Part 3: Warfare</b>			
3	Tennis modelling	1.	3.	1./2.
4	Warfare strategies	1.	3.	3.
	<b>Part 4: Two-person zero-sum games</b>			
5	Dominance, Saddle points, Mixed strategies, Minimax theorem, Utilities	1./2.	4./1.	7.
6	Tennis, Hierarchical games	1.	4.	
7	Poker	1./2.	4./1.	
	<b>Part 5: Two-person zero-sum games with risk</b>			
8	Risk-averse strategies, Risk of ruin, Kelly Equilibrium	1.	5.	4.
	<b>Part 6: Two-person nonzero-sum games</b>	2.	2.	
9	Nash Equilibrium, Maximin strategies, Pareto principle	2.	2.	7.
10	Prisoner's Dilemma, Strategic moves	2.	2.	7.
11	Evolutionary stable strategies	2.	2.	7.
12	Nash Arbitration Scheme, Duopoly problem	2.	2.	
13	Revision			

## Assessment

Item	Amount
Tutorial Exercises	20%
Mid-Semester Exam	40%
Semester Exam	40%

### Part 1: Risk Theory

#### Week 1 – Casino games, Kelly criterion, Video poker

Is it generally accepted that the mean and standard deviation of a random variable give sufficient characteristics for a particular distribution. Although the standard deviation is a measure of risk, it is equally important to obtain two other measures of risk namely the coefficients of skewness and kurtosis (particularly when the distribution is not a Normal Distribution). These four distributional characteristics can be applied directly to the Normal Power approximation to give the distribution of a random variable. We choose to analyze casino games since this particular form applies to lawsuits as given in chapter 2 and two-person zero-sum games when there is risk involved as given in chapter 5. A formula for the Kelly Criterion when multiple outcomes exist is derived an application is given in video poker. Also, the Kelly Criterion is applied to lawsuits and two-person zero-sum games when there is risk involved.

### Part 2: Lawsuits

#### Week 2 – Dispute resolution, Litigation, Negotiation, Arbitration

There are many decisions involving risk and uncertainty in industry. One example is whether it is worthwhile to file a lawsuit given there are risks involved from legal fees if unsuccessful in the court trial, how much to negotiate if an out-of-court settlement is a possibility and what is a “fair” arbitration value. To obtain insight to the decision-making process to this problem, a model is developed that is representative of the structure used in casino games, which utilizes the Kelly Criterion.

### Part 3: Warfare

#### Week 3 – Tennis modelling

A Markov Chain model is developed for winning a tennis match using recursion formulas in a Excel spreadsheet. This hierarchical model in tennis is used to obtain insights to optimizing resources in the hierarchical structure in warfare.

#### **Week 4 – Warfare strategies**

Four strategies are identified. Firstly, where a player can increase their effort on a particular set in a match. Which set should they increase their effort to optimize their chances of winning the match. Secondly, where a player can increase their effort on a particular set in a match and decrease their effort on a particular set in the match. Thirdly, where a player can increase their effort on ‘important’ points in a match and decrease their effort on ‘unimportant’ points in a match. And lastly, a player has a “large” number of available increases in effort for use in the match. However, there are costs associated for applying an increase in effort at a particular point (and a reward for winning the over match).

#### **Part 4: Two-person zero-sum games**

##### **Week 5: Dominance, Saddle points, Mixed strategies, Minimax theorem, Utilities**

Game theory in a general context is defined. For two-person zero-sum game, calculations are given to obtain dominance, saddle points and mixed strategies. The Minimax theorem is given along with a discussion on utility theory.

##### **Week 6: Tennis, Hierarchical games**

Applications on two-person zero-sum games are given for tennis and hierarchical games. For tennis risk-taking on serve is analysed to determine what proportion of time a player should be taking more risk on the second serve. For hierarchical games, the situation is analyzed when both players can increase their efforts on a particular set in the match.

##### **Week 7: Poker**

In matrix games we have assumed that the players make their choice of strategy simultaneously, without knowledge of what the other player is choosing. We will consider a method of modelling decisions sequentially by a game tree. We will find that, perhaps surprisingly, this new model can always be reduced to our model of a matrix game. A radically simplified version of poker is given as an example.

#### **Part 5: Two-person zero-sum games with risk**

##### **Week 8: Risk-averse strategies, Risk of ruin, Kelly Equilibrium**

One of the utility axioms states that numbers in the game matrix must be cardinal utilities and can be transformed by any positive linear function  $f(x)=ax+b$ ,  $a>0$  without changing the information they convey. However, when the payoffs for the game are monetary payoffs, it makes sense that the favourable player would be risk averse such that to reduce the probability of obtaining a negative payoff (assuming one exists). Hence risk-averse strategies are analyzed and using the Kelly Criterion which maximizes the long-term growth of the bank, the Kelly Equilibrium is devised as an alternative to the Nash Equilibrium.

## **Part 6: Two-person nonzero-sum games**

### **Week 9: Nash Equilibrium, Maximin strategies, Pareto principle**

In two-person zero-sum games the Minimax theorem gives two aspects for each player, a security level and a no regret. This is not the case with two-person nonzero-sum games. Hence the Nash Equilibrium is formalized as a no regret aspect (which also applies to zero-sum games). Maximin strategies are devised as a security level along with the Pareto principle.

### **Week 10: Prisoner's Dilemma, Strategic moves**

The Prisoner's Dilemma is one of the most widely used games in the social sciences, since it requires cooperation from both players to increase their payoff from a Nash Equilibrium outcome which is non-Pareto optimal to a "non" Nash Equilibrium that is Pareto optimal. Various strategies are analyzed to determine how cooperation can be obtained through repeated play. Strategic moves are also analyzed where commitments, threats and promises become possible.

### **Week 11: Evolutionary stable strategies**

Evolutionary stable strategies are analyzed which are particularly relevant to the study of behaviour in modern sociobiology.

### **Week 12: Nash Arbitration Scheme, Duopoly problem**

The Nash Arbitration Scheme is analyzed to determine a "fair" outcome by imagining that players called in an impartial outside arbitrator. An application to the duopoly problem is given.

### **Week 13: Revision**

## **Resources**

### Online Content

Adjusted Winner - an algorithm for fair division ([html](#))

Provably Fair Solutions ([html](#))

Fair Outcomes ([html](#))

Smartsettle ([html](#))

Game Theoretic Software ([html](#))

### Publications

Law, Probability & Risk ([html](#))

Games and Economic Behavior ([html](#))

International Journal of Game Theory ([html](#))

The RAND Journal of Economics ([html](#))

OR Insight ([html](#))  
Journal of the Operational Research Society ([html](#))  
European Journal of Operational Research ([html](#))  
ASOR Bulletin ([html](#))  
Proceedings of the International Conference on Operations Research and Enterprise Systems ([html](#))  
Australian and New Zealand Industrial and Applied Mathematics Journal ([html](#))  
Journal of the Australian Mathematical Society ([html](#))  
Journal of Conflict Resolution ([html](#))  
Public Choice ([html](#))  
Conflict Management and Peace Science ([html](#))  
Theory and Decision ([html](#))  
Group Decision and Negotiation ([html](#))  
Social Choice and Welfare ([html](#))  
Rationality and Society ([html](#))  
International Negotiation: A Journal of Theory and Practice ([html](#))  
Mathematical Social Sciences ([html](#))  
International Journal of Conflict Management ([html](#))  
Australasian Dispute Resolution Journal ([html](#))  
The International Journal of Human Resource Management ([html](#))

#### Books

Economic Fables by Ariel Rubenstein ([html](#))  
Theory of Games and Economic Behavior: 60th Anniversary Commemorative Edition by John von Neumann and Oskar Morgenstern ([html](#))  
Games and Decisions: Introduction and Critical Survey by R. Duncan Luce and Howard Raiffa ([html](#))  
Collected Papers by Robert Aumann ([html](#))  
A Beautiful Mind by Sylvia Nasar ([html](#))  
Micromotives and Macrobehavior by Thomas Schelling ([html](#))  
Prisoner's Dilemma: John von Neumann, Game Theory, and the Puzzle of the Bomb by William Poundstone ([html](#))  
Who Gets What - and Why: The New Economics of Matchmaking and Market Design by Alvin Roth ([html](#))  
The Game Theorist's Guide to Parenting: How the Science of Strategic Thinking Can Help You Deal with the Toughest Negotiators You Know-Your Kids by Paul Raeburn and Kevin Zollman ([html](#))  
Introducing Game Theory: A Graphic Guide by Ivan Pastine, Tuvana Pastine and Tom Humberstone ([html](#))  
The Joy of Game Theory: An Introduction to Strategic Thinking by Presh Talwalkar ([html](#))

Thinking Strategically: The Competitive Edge in Business, Politics, and Everyday Life by Avinash Dixit and Barry Nalebuff ([html](#))

The Art of Strategy: A Game Theorist's Guide to Success in Business and Life by Avinash Dixit and Barry Nalebuff by ([html](#))

Co-Opetition by Adam Brandenburger and Barry Nalebuff ([html](#))

Game Theory: An Introduction by Steven Tadelis ([html](#))

Game Theory 101: The Complete Textbook by William Spaniel ([html](#))

Insights into Game Theory: An Alternative Mathematical Experience by Ein-Ya Gura ([html](#))

Rock, Paper, Scissors: Game Theory in Everyday Life by Len Fisher ([html](#))

Game Theory at Work: How to Use Game Theory to Outthink and Outmaneuver Your Competition by James Miller ([html](#))

Game Theory 101: Bargaining by William Spaniel ([html](#))

The Game Theory by Uri Bram ([html](#))

Game Theory: A Very Short Introduction by Ken Binmore ([html](#))

Gaming the Vote: Why Elections Aren't Fair (and What We Can Do About It) by William Poundstone ([html](#))

Game-Changer: Game Theory and the Art of Transforming Strategic Situations by David McAdams ([html](#))

Game Theory Evolving: A Problem-Centered Introduction to Modeling Strategic Interaction - Second Edition by Herbert Gintis ([html](#))

Playing for Real: A Text on Game Theory by Ken Binmore ([html](#))

Game Theory by Drew Fudenberg and Jean Tirole ([html](#))

Games and Information: An Introduction to Game Theory by Eric Rasmusen ([html](#))

Putting Auction Theory to Work (Churchill Lectures in Economics) by Paul Milgrom ([html](#))

Game Theory: Analysis of Conflict by Roger Myerson ([html](#))

Game Theory: A Non-technical Introduction by Mortin Davis ([html](#))

Micromotives and Macrobehavior by Thomas Schelling ([html](#))

The Strategy of Conflict: With a New Preface by the Author by Thomas Schelling ([html](#))

The Compleat Strategyst: Being a Primer on the Theory of Games of Strategy by John Williams ([html](#))

Two-Person Game Theory by Anatol Rapoport ([html](#))

Games of Strategy: Theory and Applications by Melvin Dresher ([html](#))

The Essential John Nash edited by Harold Kuhn and Sylvia Nasar ([html](#))

Freakonomics: A Rogue Economist Explores the Hidden Side of Everything by Steven Levitt and Stephen Dubner ([html](#))

SuperFreakonomics: Global Cooling, Patriotic Prostitutes, and Why Suicide Bombers Should Buy Life Insurance by Steven Levitt and Stephen Dubner ([html](#))

Think Like a Freak: The Authors of Freakonomics Offer to Retrain Your Brain by Steven Levitt and Stephen Dubner ([html](#))

Jane Austen, Game Theorist by Michael Suk-Young Chwe ([html](#))

The Prisoner's Dilemma by Sean Stuart O'Connor ([html](#))



Getting to Yes: Negotiating Agreement Without Giving by Roger Fisher, William Ury and Bruce Patton ([html](#))

On Numbers and Games by John Conway ([html](#))

The Selfish Gene: 30th Anniversary Edition by Richard Dawkins ([html](#))

Liars and Outliers: Enabling the Trust that Society Needs to Thrive by Bruce Schneier ([html](#))

Negotiauctions: New Dealmaking Strategies for a Competitive Marketplace by Guhan Subramanian ([html](#))

The Survival Game by David Barash ([html](#))

The Logic of Life: The Rational Economics of an Irrational World by Tim Harford ([html](#))

It's Not You, It's the Dishes: How to Minimize Conflict and Maximize Happiness in Your Relationship by Paula Szuchman and Jenny Anderson ([html](#))

Critical Mass: How One Thing Leads to Another by Philip Ball ([html](#))

The Social Atom: Why the Rich Get Richer, Cheaters Get Caught, and Your Neighbor Usually Looks Like You by Mark Buchanan ([html](#))

The Predictioneer's Game: Using the Logic of Brazen Self-Interest to See and Shape the Future by Bruce Bueno De Mesquita ([html](#))

Priceless: The Myth of Fair Value (and How to Take Advantage of It) by William Poundstone ([html](#))

Game Theory 101: The Rationality of War by William Spaniel ([html](#))

SuperCooperators: Altruism, Evolution, and Why We Need Each Other to Succeed by Martin Nowak and Roger Highfield ([html](#))

Nonzero: The Logic of Human Destiny by Robert Wright ([html](#))

Game Theory and Politics by Steven Brams ([html](#))

Biblical Games: Game Theory and the Hebrew Bible by Steven Brams ([html](#))

Game Theory and the Humanities: Bridging Two Worlds by Steven Brams ([html](#))

Fair Division: From Cake-Cutting to Dispute Resolution by Steven Brams ([html](#))

The Win-Win Solution: Guaranteeing Fair Shares to Everybody by Steven Brams and Alan Taylor ([html](#))

Theory of Moves by Steven Brams ([html](#))

Approval Voting by Steven Brams and Peter Fishburn ([html](#))

Mathematics and Democracy: Designing Better Voting and Fair-Division Procedures by Steven Brams ([html](#))

Divine Games: Game Theory and the Undecidability of a Superior Being by Steven Brams ([html](#))

The Presidential Election Game by Steven Brams ([html](#))

Negotiation Games: Applying Game Theory to Bargaining and Arbitration by Steven Brams ([html](#))

Paradoxes in Politics: An Introduction to the Nonobvious in Political Science by Steven Brams ([html](#))

Spatial Models Of Election Competition by Steven Brams ([html](#))

The Mathematics Of Preference, Choice And Order: Essays In Honor Of Peter C. Fishburn edited by Steven Brams ([html](#))

Rational Politics: Decisions, Games, and Strategy by Steven Brams ([html](#))

Superpower Games: Applying Game Theory To Superpower Conflict by Steven Brams ([html](#))

Game Theory and the Law by Douglas Baird, Robert Gertner and Randal Picker ([html](#))

Beautiful Game Theory: How Soccer Can Help Economics by Ignacio Palacios-Huerta ([html](#))

Behavioral Game Theory: Experiments in Strategic Interaction by Colin Camerer ([html](#))

Security and Game Theory: Algorithms, Deployed Systems, Lessons Learned by Milind Tambe ([html](#))

Game Theory by Michael Maschler, Eilon Solan and Shmuel Zamir([html](#))

Networks, Crowds, and Markets: Reasoning about a Highly Connected World by David Easley and Jon Kleinberg ([html](#))

Game Theory, Alive by Anna Karlin and Yuval Peres ([pdf](#))

Algorithmic Game Theory edited by Noam Nisan, Tim Roughgarden, Eva Tardos and Vijay Vazirani ([html](#))

Language of Game Theory, The: Putting Epistemics into the Mathematics of Games by Adam Brandenburger ([html](#))

Game Theory in Action: An Introduction to Classical and Evolutionary Models by Stephen Schecter and Herbert Gintis ([html](#))

Games of Strategy by Avinash Dixit, Susan Skeath and David Reiley Jr. ([html](#))

Game Theory: An open access textbook with 165 solved exercises by Giacomo Bonanno ([pdf](#))

Game Theory and Climate Change by Parkash Chander ([html](#))

Game Theory Through Examples by Erich Prisner ([pdf](#))

Game Theory Evolving: A Problem-Centered Introduction to Modeling Strategic Interaction by Herbert Gintis ([html](#))

Games and Information: An Introduction to Game Theory by Eric Rasmusen ([html](#))

Game Theory by Michael Maschler, Eilon Solan and Shmuel Zamir ([html](#))