How tennis, probabilistic games and game theory examples could be used for teaching primary and secondary level mathematics

By
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Abstract

Students enjoy playing games. Therefore, it seems logical to use games that have a mathematical basis in order to enhance the understanding of mathematical concepts for teaching primary and secondary level mathematics. Similarly, students enjoy watching sport (tennis in particular), and the mathematics of tennis can also be used for teaching primary and secondary level mathematics. Tennis also allows students to built their own tennis calculator in an Excel spreadsheet (hands-on experience) and can use their own calculator to understand mathematical concepts during a live tennis match. Differential calculus is a key concept in the later years of high school mathematics and can be applied to tennis.

Keywords: mathematics; tennis; game theory, probabilistic games; gambling

1. Introduction

Tennis is the fifth most popular sport in the world with 1 billion fans (behind field hockey, basketball, cricket and soccer). But according to TIME tennis is the most popular women's sport. So accounting for both genders, tennis could be considered the most popular sport. Probability theory is a key underlying concept is primary school mathematics education. Primary school students can learn the mathematics of tennis through counting paths (p1 'The Mathematics of Tennis') and build a simple Excel spreadsheet of a tennis game using built-in absolute and relative referencing formulas (hands-on experience) and hence obtain the chances of players winning a game (p17 'The Mathematics of Tennis'), and extend this to the chances of winning a set and a match conditional on the scoreboard, and hence use their calculator to develop the understanding of mathematical probabilities during an actual tennis match as a classroom exercise. OnCourt tennis software can be used to obtain the initial parameters for the tennis model (chances of players winning a point on serve) and post-match statistics from OnCourt can be used to calculate the actual percentages that players won on serve (and compared to the estimated pre-match percentage of players winning a point on serve).

Primary school students enjoy playing games. So it makes sense for students to engage in games that have a mathematical basis particularly in probability theory. This could include playing a toy roulette wheel, blackjack, backgammon, le her, poker and video poker. An explanation of the percent house margin in roulette could be explained and compared to the simulated results. Basic strategies in blackjack could be explained along with the percent house margin with perfect basic strategy play (refer to The Wizard of Odds), and students

could watch the movie “21” during class to have an appreciation on how with a high level of sophistication a gambling game can be profitable. Backgammon (refer to GNU Backgammon) and video poker (refer to Zamzow software solutions) probabilities could be explained and students could play this software during class. Optimal mixed strategies in le her and a simplified poker game (refer to Game Theory and Strategy by Philip Straffin) could be explained as a way of introducing game theory concepts. Also, the game paper, scissors and rock is popular amongst primary school students and can be explained using game theory to obtain optimal strategies.

High school students can extend their Excel spreadsheet (if not successful at a primary level) to obtain the chances of winning a tiebreak game, set and match; along with the chances of winning from any score line within the match, and hence a obtain a calculator similar to Interactive Java Tennis Calculator (refer to Chapter 2 ‘The Mathematics of Tennis’). Students could also learn backwards and forwards recursion concepts in mathematics. OnCourt allows you to compare serving and receiving statistics conditional on the court surface for head-to-head and across all players, and hence engage students in statistical knowledge. OnCourt also provides match statistics at the completion of each match which can be used in teaching for understanding probability theory at a more advanced level compared to primary school mathematics (Barnett 2010). A comprehensive tennis calculator has been developed in Excel (Tennis Calculator) based on recurrence formulas (no programming) and students could further their tennis calculator by developing the mean and variance of the number of points in a game, set and match and hence obtain an understanding of such concepts. The Binomial theorem (and Pascals triangle) can be applied in tennis to obtain the chances of winning a game as well as the distribution of points played in a game. The chances of players winning from deuce in a game can be obtained by summing an infinite geometric series. Also, Markov Chain theory could be introduced by calculating the probabilities of winning a game. The four familiar characteristics of distribution of the number of points in a match can be applied to the Normal Distribution (or even better the Normal Power approximation) to introduce students to such concepts.

An underlying concept of high school mathematics is differential calculus (or anti-differentiation known as integration). Also, probability theory is an important concept. Calculating the four common distributional characteristics (mean, variance, coefficients of skewness and kurtosis) for the number of points remaining in a game from deuce requires using the moment generating function which uses differential calculus.

Understanding the mathematics of gambling allows high school students to become sensible consumers in this risk-taking activity (yet high schools are looking to ban gambling from teaching). The fundamentals of a casino game could be taught (refer to Resolving Problem Gambling: a mathematical approach). Also, students would have an appreciation for gambling with the World Poker Tour being an internationally televised gaming and entertainment brand since 2002. Students could watch the World Poker Tour during class and calculate the probabilities of poker hands which used permutations and combinations (refer to The Theory of Gambling and Statistical Logic by Richard Epstein), and use software such as CardPlayer.com to let you run any scenario that you see at the poker table, see your odds and outs, and cover the math of winning and losing poker hands. Two-person zero-sum games could be taught to introduce game theory where a simplified poker game is given in Game

Theory and Strategy by Philip Straffin. Progressive video poker is mathematically well defined where the odds are actually in the players favour. Analysing probabilities through Zamzow Software Solutions could enhance students understanding of probability theory in mathematics. Card-counting in blackjack is interesting to analyse to demonstrate how the odds change depending on the deck composition of cards (dependent trials) and how one can obtain an advantage over the house. This was depicted by the movie "21" and hence could be used as a mathematics teaching tool where the game of blackjack is analysed by watching the movie "21" during class. Students can also play backgammon and bridge during class and learn probabilities and in particular permutations and combinations in bridge (refer to The Theory of Gambling and Statistical Logic by Richard Epstein). Chess could also be played in class to engage students in a logic way of thinking which could potentially improvement mathematical knowledge.

2. Primary school probability theory through tennis

Excel spreadsheet code to obtain the conditional probabilities of player A winning a game on serve is as follows:

Enter the text p_A (probability of player A winning a point on serve) in cell D1.
Enter the text q_A (=1- p_A) in cell D2
Enter 0.6 in cell E1
Enter =1-E1 in cell E2
Enter 1 in cells C11, D11 and E11
Enter 0 in cells G7, G8 and G9
Enter = E1^2/(E1^2+E2^2) in cell F10
Enter =$E$1*C8+$E$2*D7 in cell C7
Copy and Paste cell C7 in cells D7, E7, F7, C8, D8, E8, F8, C9, D9, E9, F9, C10, D10 and E10

By changing the value in cell E1 (on the probability of player A winning a point on serve) gives the chances of player A winning a game on serve

Similar spreadsheet code can be given to calculate the chances of player B winning a game on serve, winning a tiebreak game, a set and a match; and hence the probabilities of players winning a match can be obtained conditional on the scoreboard. Primary school students can use this calculator whilst watching a tennis match in class as a way of obtaining insights to probabilities in mathematics. The probabilities of players winning a game, set and match conditional on the scoreboard can also be obtained from an Interactive Java Tennis Calculator. OnCourt tennis software can be used to obtain the parameters for the model (tennis calculator) as a constant probability of each player winning a point a serve. OnCourt also provides match statistics at the completion of each match which can be used in teaching for understanding probability theory (Barnett, 2010). For example, the probabilities of players winning a point on serve can be obtained from a simple formula

Winning Percentage on serve = 1^{st} serve % * Winning % on 1^{st} serve + (1-1^{st} serve %) * Winning % on 2^{nd} serve

Alternatively,
Winning Percentage on serve = 1- Receiving Points Won (for the opponent)

And hence the two equations should be equivalent.

3. Secondary school probability theory through tennis

Recurrence formulas to obtain numerical results is the best approach to modelling outcomes in a tennis match as they can be readily implemented on spreadsheets such as Excel (including primary school students as demonstrated above).

Let p_A represent a constant probability of player A winning a point on serve
Let q_A represent a constant probability of player A losing a point on serve (player B winning a point on player A's serve)

It follows that $q_A=1-p_A$

Let $P_A(a,b)$ represent the probability that player A wins the game on serve when the score is (a,b)

The backward recursion formula becomes:
$P_A(a,b) = p_AP_A(a + 1,b) + q_AP_A(a,b + 1)$

The boundary values are:
$P_A(a,b) = 1$, if $a = 4$ and $b\leq 2$
$P_A(a,b) = 0$, if $b = 4$ and $a\leq 2$
$P_A(3,3) = p_A^2/(p_A^2+q_A^2)$

The boundary value $P_A(3,3)$ can be calculated as a sum of a geometric infinite series
$P_A(3,3) = p_A^2 + p_A^2(2p_Aq_A) + p_A^2(2p_Aq_A)^2 + p_A^2(2p_Aq_A)^3 +\ldots\ldots .$
where the first term is p_A^2 and the common ratio is $2p_Aq_A$

Therefore $P_A(3,3) = p_A^2 / (1-2p_Aq_A)$
This can be simplified to $P_A(3,3) = p_A^2/(p_A^2+q_A^2)$
Refer to p16 in 'The Mathematics of Tennis' for more detail for this calculation.

The number of points played in a game of tennis and the number of points remaining in the game of tennis have a shift in mean M but the variance V (and coefficients of skewness and kurtosis) remain unchanged as given by the same shape of both distributions. This is given by the mean and variance properties.

Let $X_A(a,b)$ and $Y_A(a,b)$ be random variables of the total number of points played in a game and the number of points remaining in a game respectively at point score (a,b) for player A serving.

$M(X+c)=M(X)+c$
$V(X+c)=V(X)$

for c: constant

The graphical representation is given in p60 in ‘The Mathematics of Tennis’.

Secondary school differential calculus through tennis

Calculating the four common distributional characteristics (mean, variance, coefficients of skewness and kurtosis) for the number of points remaining in a game from deuce requires using the moment generating function which uses differential calculus. The mathematics is given in p61 in ‘The Mathematics of Tennis’.

4. Secondary school analysis of casino games

Assuming independent trials, a casino game is a set of outcomes (profit payouts) with associated probabilities. This information allows you to obtain the first four moments which can be used to calculate familiar characteristics of a distribution after N bets (mean, standard deviation, coefficients of skewness and kurtosis). Hence with two outcomes the binomial formula can be used to calculate the exact distribution after N bets (refer to The Mathematics of Games and Gambling by Edward Packel), otherwise the Normal Distribution (or even better the Normal Power approximation) can be applied. The mathematics is given in p11 in ‘Resolving Problem Gambling: a mathematical approach’.

5. Secondary school game theory through poker

Consider the following radically simplified version of a game of poker. Each of two players, Player 1 and Player 2, puts $1 into the pot as “ante”. Each is then dealt a hand, which consists of one card, from a large deck which consists only of aces and kings. Player 2 must decide whether to bet $2 or to drop. If he drops, Player 1 wins the pot. If Player 2 bets, Player 1 must decide whether to call by matching Player 2’s bet, or to fold. If Player 1 folds, Player 2 wins the pot. If Player 1 calls, the players compare their hands and the higher card wins the pot. If the hands tie, the pot is split equally. The mathematics is given in Game Theory and Strategy by Philip Straffin.

6. Conclusions

This article has used examples from tennis, probabilistic games and game theory to teach mathematical concepts at a primary and secondary level mathematics education. In tennis, students can build their own tennis calculator using built-in functions in Excel spreadsheets and apply this calculator to enhance the understanding of probability theory during a tennis match, and hence watching a live tennis match during class as a way of teaching mathematical concepts. Similarly, the movie “21” is based on the game of blackjack and hence watching this movie during class and understanding the mathematical concepts behind the game, is a way of teaching mathematics. Students can also learn mathematical concepts by playing games of video poker, le her, bridge, backgammon and poker. Differential calculus is a key concept in the later years of high school mathematics and can be applied to tennis.

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