Duration of a game of tennis – using forward recursion

Let $X(a,b)$ be a random variable of the total number of points played in a game at point score (a, b). Let $f(X(a,b)=x(a,b))$ represent the distribution of the total number of points played in the game at point score (a,b). Let $N(g,h|a, b)$ be the probability of reaching a point score (g,h) in a game from point score (a,b), where a and b represent the current point score for the server and receiver respectively, and g and h represent the projected point score for the server and receiver respectively.

The distribution of the total number of points played in a game at point score (a,b) is represented by:

$f(X(a,b)=4)=N(4,0|a,b)+ N(0,4|a,b)$
$f(X(a,b)=5)=N(4,1|a,b)+ N(1,4|a,b)$
$f(X(a,b)=6)=N(4,2|a,b)+ N(2,4|a,b)$

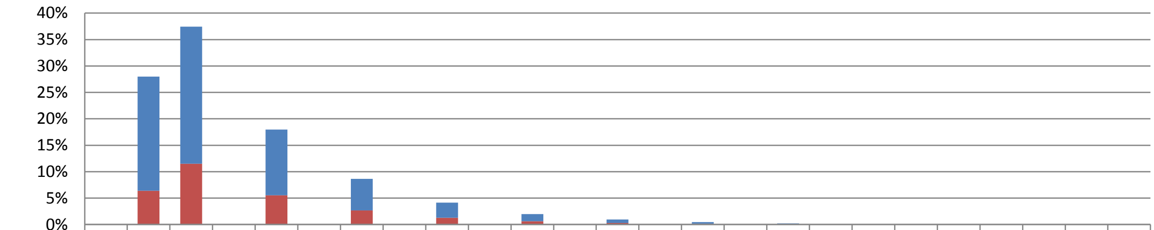
$f(X(a,b)=x(a,b))=N(\frac{x(a,b)+2}{2}, \frac{x(a,b)-2}{2}|a,b) +N(\frac{x(a,b)-2}{2}, \frac{x(a,b)+2}{2}|a,b)$ for $x(a,b)=8,10,12,\ldots.$

Let $Y(a,b)$ be a random variable of the number of points remaining in a game at point score (a,b). Let $f(Y(a,b)=y(a,b))$ represent the distribution of the number of points remaining in the game at point score (a,b).

$f(Y(a,b)=4-a-b)=N(4,0|a,b)+ N(0,4|a,b)$
$f(Y(a,b)=5-a-b)=N(4,1|a,b)+ N(1,4|a,b)$
$f(Y(a,b)=6-a-b)=N(4,2|a,b)+ N(2,4|a,b)$
$f(Y(a,b)=y(a,b)-a-b)=N(\frac{y(a,b)+2)}{2}, \frac{y(a,b)-2}{2}|a, b)+N(\frac{y(a,b)-2)}{2}, \frac{y(a,b)+2}{2}|a, b)$, for $y(a,b)=8,10,12,\ldots.$

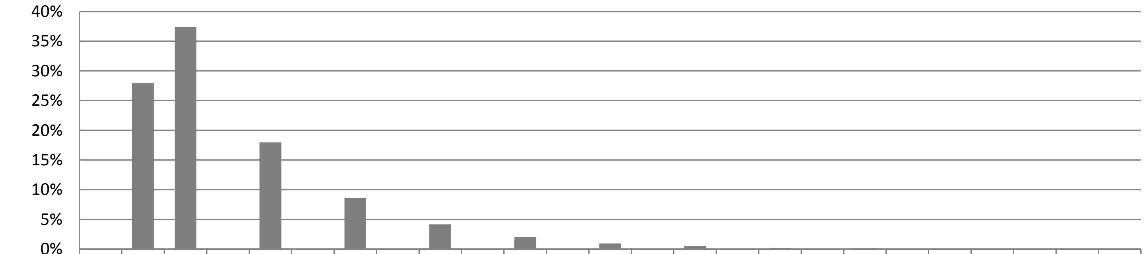
Note the relation $f(X(0,0)=x(0,0)) = f(Y(0,0)=y(0,0))$, for all $x(0,0)=y(0,0)$. This implies that from the outset the distribution of the total number of points played in the game is equivalent to the distribution of the number of points remaining in the game.

Let p represent the probability of the server winning a point. Figures 1 and 2 represent the distributions of the total number of points played in a game and the number of points remaining in a game respectively from $a = 1$, $b = 1$ for the server with $p = 0.60$. Note that the shape of both distributions are the same. In other words the variance remains unchanged by adding a constant to all values of the variable. This is widely known as an invariant property in variance such that $V(X +a) = V(X)$.



	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Player A	0.0%	21.6	25.9	0.0%	12.4	0.0%	6.0%	0.0%	2.9%	0.0%	1.4%	0.0%	0.7%	0.0%	0.3%	0.0%	0.2%	0.0%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Player B	0.0%	6.4%	11.5	0.0%	5.5%	0.0%	2.7%	0.0%	1.3%	0.0%	0.6%	0.0%	0.3%	0.0%	0.1%	0.0%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

Figure 1: The distribution of the total number of points played in a game from *a*=1, *b*=1 with *p*=0.60



	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
Points	0.0%	28.0	37.4	0.0%	18.0	0.0%	8.6%	0.0%	4.1%	0.0%	2.0%	0.0%	1.0%	0.0%	0.5%	0.0%	0.2%	0.0%	0.1%	0.0%	0.1%	0.0%	0.0%	0.0%	0.0%

Figure 2: The distribution of the number of points remaining in a game from a=1, b=1 with *p*=0.60