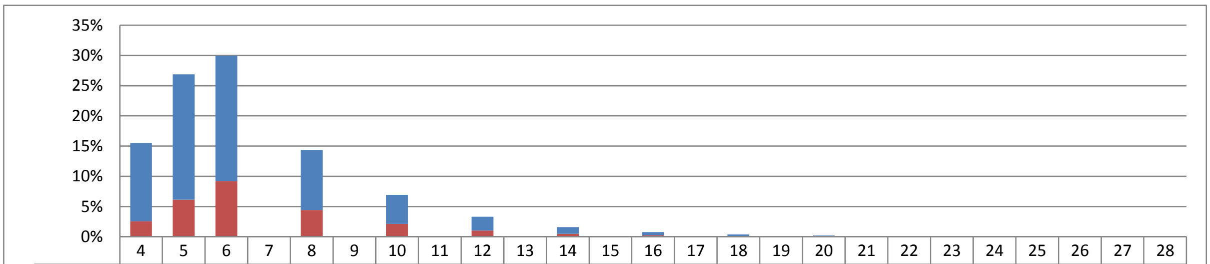
Duration of a game of tennis: using the Binomial theorem

Suppose we wish to determine the distribution of the total number of points played in a game from the outset; that is: "What is the probability of playing 4,5,6,8,10.....points?". Let p represent the probability of the server winning a point and q=1-p. It was established in[1] using the Binomial theorem, that the probability of the server winning the game to 0 on serve is given by p^4. Similarly the probability of the receiver winning the game to 0 is q^4. This implies that the probability of playing exactly 4 points in the game is given p^4+q^4 (since 4 points are required for either server or receiver to win the game to 0). Similarly, the probability of playing exactly 5 points in the game is $4p^4q + 4q^4p$. Similarly the probability of playing exactly 6 points in the game is $10p^4q^2+10q^4p^2$. The probability of reaching deuce was obtained as[1] $20p^3q^3$. Therefore the probability of playing exactly 8 points is obtained as $20p^3q^3\ (p^2+q^2)$. Similarly the probability of playing exactly 10 points is obtained as $20p^3q^3(p^2+q^2)(2pq)$. Similarly the probability of playing exactly 12 points is obtained as $20p^3q^3(p^2+q^2)(2pq)^2$. Therefore the probability of playing x points (x=8,10,12,14...) is obtained as $20p^3q^3(p^2+q^2)(2pq)^{(x-8)/2}$.

More formally, let X be a random variable of the total number of points played in a game from the outset. Let $f(X=x)$ represent the distribution of the total number of points played in the game from the outset for the server. This distribution is given as follows.

$f(X=4)= p^4+q^4$
$f(X=5)= 4p^4q + 4q^4p$
$f(X=6)= 10p^4q^2+10q^4p^2$
$f(X=x)= 20p^3q^3(p^2+q^2)(2pq)^{(x-8)/2}$, if x=8,10,12,…

Figure 1 represents the distribution graphically of the total number of points played in a game from the outset for p=0.60. Notice how the blue colour is the chances of the server winning the game and the maroon colour is the chances of the receiver winning the game. For example, the chances of the server winning the game to 15 is given by the frequency distribution of blue for 5 total points played. This numerical value is 20.74%. Similarly, the chances of the receiver winning the game to 15 is given by the frequency distribution of maroon for 5 total points played. This numerical value is 6.14%. Therefore, the game finishing with either player winning to 15 (or 5 total points played) is given by 20.74% + 6.14% = 26.9%.



	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Player A	13.0	20.7	20.7	0.0%	10.0	0.0%	4.8%	0.0%	2.3%	0.0%	1.1%	0.0%	0.5%	0.0%	0.3%	0.0%	0.1%	0.0%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Player B	2.6%	6.1%	9.2%	0.0%	4.4%	0.0%	2.1%	0.0%	1.0%	0.0%	0.5%	0.0%	0.2%	0.0%	0.1%	0.0%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

Figure 1: The distribution of the total number of points played in a game from the outset for p=0.60

Reference

[1]Barnett T. Winning a game of tennis: using the Binomial theorem. To appear in JMST