Applying the Normal Power approximation to a tennis set

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The aim is to apply the Normal Power approximation to a tiebreak and an advantage set to obtain the distribution of the number of points played in a set. For increased accuracy it is necessary to add the situation prior to 6 games all in a set to the situation from 6 games all in the set. The recursive formulas are similar to 'A recursive approach to modelling the amount of time played in a tennis match'. The differences occur in the boundary conditions.

Tiebreak set

Points in a set (up to 6-6)

$P^{gsT}(c,d|s_A,w_A,n_A)=P^{gsT}(c,d|s_A,w_A,n_B)=P^{gsT}(c,d|s_A,l_A,n_A)=P^{gsT}(c,d|s_A,l_A,n_B)=0$, if (c,d)=(6,6)

$w_n(Y^{psT}(c,d|s_A,w_A,n_A))=w_n(Y^{psT}(c,d|s_A,w_A,n_B))=w_n(Y^{psT}(c,d|s_A,l_A,n_A))=w_n(Y^{psT}(c,d|s_A,l_A,n_B))=$
$w_n(Y^{psT}(c,d|s_B,w_B,n_A))=w_n(Y^{psT}(c,d|s_B,w_B,n_B))=w_n(Y^{psT}(c,d|s_B,l_B,n_A))=w_n(Y^{psT}(c,d|s_B,l_B,n_B))=0$, if (c,d)=(0,0)

Points in a set (from 6-6)

$P^{gsT}(c,d|s_A,w_A,n_A)=P^{gsT}(c,d|s_A,w_A,n_B)=P^{gsT}(c,d|s_A,l_A,n_A)=P^{gsT}(c,d|s_A,l_A,n_B)=0$, if a=6 and 0≤4≤2; b=6 and 0≤a≤4; (7,5); (5,7)

$w_n(Y^{psT}(c,d|s_A,w_A,n_A))=w_n(Y^{psT}(c,d|s_A,w_A,n_B))=w_n(Y^{psT}(c,d|s_A,l_A,n_A))=w_n(Y^{psT}(c,d|s_A,l_A,n_B))=$
$w_n(Y^{psT}(c,d|s_B,w_B,n_A))=w_n(Y^{psT}(c,d|s_B,w_B,n_B))=w_n(Y^{psT}(c,d|s_B,l_B,n_A))=w_n(Y^{psT}(c,d|s_B,l_B,n_B))$, 0 if a=6 and 0≤4≤2; b=6 and 0≤a≤4; (7,5); (5,7)

Parameters

Parameters	Up to 6-6	From 6-6
P	0.707	0.293
μ	27.180	50.114
σ	8.766	6.923
γ_1	0.806	0.819
γ_2	0.323	1.033

Characteristics of distribution of a tiebreak set for p_A=0.62 and p_B=0.60 conditional on score (c,d)=(3,3)

x	z=(x- μ)/σ	y=NP	F_1= NORMSDIST(y)*P
40	1.46	1.37	0.647
41	1.58	1.47	0.657
42	1.69	1.58	0.667
43	1.80	1.69	0.675
44	1.92	1.81	0.682
45	2.03	1.93	0.688

46	2.15	2.06	0.693
47	2.26	2.19	0.697
48	2.37	2.33	0.700
49	2.49	2.48	0.703

Points in a tiebreak set up to 6-6 conditional on the game score (3,3) for p_A=0.62 and p_B=0.60

Let $NP=z-1/6\gamma_1(z^2-1)-(1/24)\gamma_2(z^3-3z)+(1/36)(\gamma_1^2)(4z^3-7z)$

x	$z=(x-\mu)/\sigma$	y=NP	F_2= NORMSDIST(y)*P
40	-1.46	-1.71	0.013
41	-1.32	-1.49	0.020
42	-1.17	-1.27	0.030
43	-1.03	-1.07	0.042
44	-0.88	-0.87	0.056
45	-0.74	-0.69	0.072
46	-0.59	-0.51	0.089
47	-0.45	-0.34	0.107
48	-0.31	-0.18	0.125
49	-0.16	-0.03	0.143

Points in a tiebreak set from 6-6 conditional on the game score (3,3) for p_A=0.62 and p_B=0.60

x	$F=F_1+F_2$	1-F	Points	Prob
0	0.000	1.000	50	0.135
1	0.000	1.000	100	0.000
2	0.000	1.000	150	0.000
3	0.000	1.000	200	0.000
4	0.000	1.000	250	0.000
5	0.000	1.000	300	0.000
6	0.000	1.000	350	0.000
7	0.000	1.000	400	0.000
8	0.001	0.999	450	0.000
9	0.001	0.999	500	0.000
10	0.003	0.997	550	0.000
11	0.006	0.994	600	0.000
12	0.011	0.989	650	0.000
13	0.018	0.982	700	0.000
14	0.029	0.971	750	0.000
15	0.043	0.957	800	0.000
16	0.060	0.940	850	0.000
17	0.081	0.919	900	0.000
18	0.106	0.894	950	0.000
19	0.133	0.867	1000	0.000
20	0.163	0.837	1050	0.000
21	0.195	0.805	1100	0.000
22	0.227	0.773	1150	0.000
23	0.260	0.740	1200	0.000
24	0.293	0.707	1250	0.000
25	0.325	0.675		
26	0.356	0.644		

27	0.386	0.614		
28	0.415	0.585		
29	0.442	0.558		
30	0.468	0.532		
31	0.492	0.508		
32	0.515	0.485		
33	0.536	0.464		
34	0.556	0.444		
35	0.575	0.425		
36	0.592	0.408		
37	0.609	0.391		
38	0.626	0.374		
39	0.642	0.358		
40	0.659	0.341		
41	0.677	0.323		
42	0.697	0.303		
43	0.717	0.283		
44	0.738	0.262		
45	0.760	0.240		
46	0.782	0.218		
47	0.804	0.196		
48	0.826	0.174		
49	0.846	0.154		
50	0.865	0.135		

Probability of a tiebreak set going beyond a specified number of points conditional on the game score (3,3) for p_A=0.62 and p_B=0.60

Advantage set

Points in a set (up to 6-6)

$P^{gs}(c,d|s_A,w_A,n_A)=P^{gs}(c,d|s_A,w_A,n_B)=P^{gs}(c,d|s_A,l_A,n_A)=P^{gs}(c,d|s_A,l_A,n_B)=0$, if (c,d)=(6,6)

$w_n(Y^{ps}(c,d|s_A,w_A,n_A))=w_n(Y^{ps}(c,d|s_A,w_A,n_B))=w_n(Y^{ps}(c,d|s_A,l_A,n_A))=w_n(Y^{ps}(c,d|s_A,l_A,n_B))= w_n(Y^{ps}(c,d|s_B,w_B,n_A))=w_n(Y^{ps}(c,d|s_B,w_B,n_B))=w_n(Y^{ps}(c,d|s_B,l_B,n_A))=w_n(Y^{ps}(c,d|s_B,l_B,n_B))=0$, if (c,d)=(0,0)

Points in a set (from 6-6)

$P^{gs}(c,d|s_A,w_A,n_A)=P^{gs}(c,d|s_A,w_A,n_B)=P^{gs}(c,d|s_A,l_A,n_A)=P^{gs}(c,d|s_A,l_A,n_B)=0$, if a=6 and $0\le 4\le 2$; b=6 and $0\le a\le 4$; (7,5); (5,7)

$w_n(Y^{ps}(c,d|s_A,w_A,n_A))=w_n(Y^{ps}(c,d|s_A,w_A,n_B))=w_n(Y^{ps}(c,d|s_A,l_A,n_A))=w_n(Y^{ps}(c,d|s_A,l_A,n_B))= w_n(Y^{ps}(c,d|s_B,w_B,n_A))=w_n(Y^{ps}(c,d|s_B,w_B,n_B))=w_n(Y^{ps}(c,d|s_B,l_B,n_A))=w_n(Y^{ps}(c,d|s_B,l_B,n_B))$, 0 if a=6 and $0\le 4\le 2$; b=6 and $0\le a\le 4$; (7,5); (5,7)

Parameters

Parameters	Up to 6-6	From 6-6
P	0.707	0.293
μ	27.180	73.022
σ	8.766	28.603
γ_1	0.806	1.896
γ_2	0.323	5.558

Characteristics of distribution of an advantage set for p_A=0.62 and p_B=0.60 conditional on score (c,d)=(3,3)

x	z=(x- μ)/σ	y=NP	F_1= NORMSDIST(y)*P
40	1.46	1.37	0.647
41	1.58	1.47	0.657
42	1.69	1.58	0.667
43	1.80	1.69	0.675
44	1.92	1.81	0.682
45	2.03	1.93	0.688
46	2.15	2.06	0.693
47	2.26	2.19	0.697
48	2.37	2.33	0.700
49	2.49	2.48	0.703

Points in an advantage set up to 6-6 conditional on the game score (3,3) for p_A=0.62 and p_B=0.60

F_2=if(z<0,NORMSDIST(y)*P,NORMSDIST(γ_1/6)+(1-EXP(-z))(P-μ)

x	z=(x- μ)/σ	y=NP	F_2
40	-1.15	-1.51	0.02
41	-1.12	-1.43	0.02
42	-1.08	-1.35	0.03
43	-1.05	-1.27	0.03
44	-1.01	-1.19	0.03
45	-0.98	-1.12	0.04
46	-0.94	-1.05	0.04
47	-0.91	-0.98	0.05
48	-0.87	-0.91	0.05
49	-0.84	-0.84	0.06

Points in a advantage set from 6-6 conditional on the game score (3,3) for p_A=0.62 and p_B=0.60

x	F=F_1+F_2	1-F	Points	Prob
0	0.000	1.000	50	0.232
1	0.000	1.000	100	0.043
2	0.000	1.000	150	0.007
3	0.000	1.000	200	0.001
4	0.000	1.000	250	0.000
5	0.000	1.000	300	0.000
6	0.000	1.000	350	0.000
7	0.000	1.000	400	0.000
8	0.001	0.999	450	0.000
9	0.001	0.999	500	0.000

10	0.003	0.997	550	0.000
11	0.006	0.994	600	0.000
12	0.011	0.989	650	0.000
13	0.018	0.982	700	0.000
14	0.029	0.971	750	0.000
15	0.043	0.957	800	0.000
16	0.060	0.940	850	0.000
17	0.081	0.919	900	0.000
18	0.106	0.894	950	0.000
19	0.133	0.867	1000	0.000
20	0.163	0.837	1050	0.000
21	0.195	0.805	1100	0.000
22	0.227	0.773	1150	0.000
23	0.260	0.740	1200	0.000
24	0.293	0.707	1250	0.000
25	0.325	0.675		
26	0.357	0.643		
27	0.387	0.613		
28	0.416	0.584		
29	0.444	0.556		
30	0.470	0.530		
31	0.495	0.505		
32	0.518	0.482		
33	0.541	0.459		
34	0.562	0.438		
35	0.581	0.419		
36	0.600	0.400		
37	0.618	0.382		
38	0.635	0.365		
39	0.651	0.349		
40	0.666	0.334		
41	0.680	0.320		
42	0.693	0.307		
43	0.705	0.295		
44	0.716	0.284		
45	0.727	0.273		
46	0.736	0.264		
47	0.745	0.255		
48	0.754	0.246		
49	0.761	0.239		
50	0.768	0.232		

Probability of an advantage set going beyond a specified number of points conditional on the game score (3,3) for p_A=0.62 and p_B=0.60