Modelling outcomes in volleyball 2

Scoring system

Match	Best-of-5 sets
Set	First to 25 points At 24-all, 2 point lead
Final set	First to 15 points At 14-all, 2 point lead
Server in set	Winner of point
Serving first each set	Rotate server Coin toss at 2 sets-all

Chances of winning a set

Let p_A represent the probability of player A winning a point on serve
Let p_B represent the probability of player B winning a point on serve

Let $q_A=1-p_A$ and $q_B=1-p_B$

Let $P_A^{25ps}(a,b)$ and $P_B^{25ps}(a,b)$ represent the probabilities of player A winning a first to 25 point set at point score (a,b) for player A and player B serving respectively

Recurrence formulas:
$P_A^{25ps}(a,b)=p_AP_A^{25ps}(a+1,b)+q_AP_B^{25ps}(a,b+1)$
$P_B^{25ps}(a,b)=p_BP_B^{25ps}(a,b+1)+q_BP_A^{25ps}(a+1,b)$

Boundary values:
$P_A^{25ps}(a,b)= P_B^{25ps}(a,b)=1$, if $a=25$, $0\le b\le 23$
$P_A^{25ps}(a,b)= P_B^{25ps}(a,b)=0$, if $b=25$, $0\le a\le 23$
$P_A^{25ps}(24,24)=p_A^2/((1-q_Aq_B)^2-p_Aq_Ap_Bq_B)$
$P_B^{25ps}(24,24)=p_Aq_B(1+p_Ap_B-q_Aq_B)/((1-q_Aq_B)^2-p_Aq_Ap_Bq_B)$

Let $P_A^{15ps}(a,b)$ and $P_B^{15ps}(a,b)$ represent the probabilities of player A winning a first to 15 point set at point score (a,b) for player A and player B serving respectively

Recurrence formulas:
$P_A^{15ps}(a,b)=p_AP_A^{15ps}(a+1,b)+q_AP_B^{15ps}(a,b+1)$
$P_B^{15ps}(a,b)=p_BP_B^{15ps}(a,b+1)+q_BP_A^{15ps}(a+1,b)$

Boundary values:
$P_A^{15ps}(a,b)= P_A^{15ps}(a,b)=1$, if $a=15$, $0\le b\le 13$
$P_B^{15ps}(a,b)= P_B^{15ps}(a,b)=0$, if $b=15$, $0\le a\le 13$
$P_A^{15ps}(14,14)=p_A^2/((1-q_Aq_B)^2-p_Aq_Ap_Bq_B)$
$P_B^{15ps}(14,14)=p_Aq_B(1+p_Ap_B-q_Aq_B)/((1-q_Aq_B)^2-p_Aq_Ap_Bq_B)$

	25-point set		15-point set	
p_A, p_B	A serving	B serving	A serving	B serving
0.30, 0.30	0.48	0.52	0.47	0.53
0.30, 0.29	0.51	0.56	0.49	0.56
0.30, 0.25	0.63	0.69	0.59	0.66
0.25, 0.25	0.47	0.53	0.46	0.54
0.25, 0.24	0.50	0.57	0.48	0.57
0.20, 0.20	0.46	0.54	0.45	0.55

Table: The probability of team A winning a first to 25 points and first to 15 points set for different values of p_A and p_B from the start of the set

Chances of winning a match

Let $P_A^{25ps}(0,0) = p_A^{25s}$
Let $P_B^{25ps}(0,0)=q_B^{25s}$
Let $P_A^{15ps}(0,0) = p_A^{15s}$
Let $P_B^{15ps}(0,0)=q_B^{15s}$

Let $q_A^{25s}=1- p_A^{25s}$ and $p_B^{25s}=1-q_B^{25s}$
Let $q_A^{15s}=1- p_A^{15s}$ and $p_B^{15s}=1-q_B^{15s}$

Let $P_A^{sm}(c,d)$ and $P_B^{sm}(c,d)$ represent the probabilities of player A winning a match at set score (c,d) for player A and player B serving respectively

Recurrence formulas:
$P_A^{sm}(c,d)= p_A^{25s}P_B^{sm}(c+1,d) +q_A^{25s}P_B^{sm}(c,d+1)$, if (c,d)=(0,0),(1,0),(0,1),(2,0),(0,2),(1,1)
$P_A^{sm}(c,d)= p_A^{25s}P_B^{sm}(c+1,d) +q_A^{25s}0.5(P_B^{sm}(c,d+1)+ P_A^{sm}(c,d+1))$, if (c,d)=(2,1)
$P_A^{sm}(c,d)= p_A^{25s}0.5(P_B^{sm}(c+1,d)+P_A^{sm}(c+1,d)) +q_A^{25s}P_B^{sm}(c,d+1)$, if (c,d)=(1,2)
$P_B^{sm}(c,d)= q_B^{25s} P_A^{sm}(c+1,d)+p_B^{25s} P_A^{sm}(c,d+1)$, if (c,d)=(0,0),(1,0),(0,1),(2,0),(0,2),(1,1)
$P_B^{sm}(c,d)= q_B^{25s} P_A^{sm}(c+1,d)+p_B^{25s} 0.5(P_A^{sm}(c,d+1)+ P_B^{sm}(c,d+1))$, if (c,d)=(2,1)
$P_B^{sm}(c,d)= q_B^{25s} 0.5(P_A^{sm}(c+1,d)+ P_B^{sm}(c+1,d))+p_B^{25s}P_A^{sm}(c,d+1)$, if (c,d)=(1,2)

Boundary values:
$P_A^{sm}(c,d)= P_B^{sm}(c,d)=1$, if (c,d)=(3,0),(3,1)
$P_A^{sm}(c,d)= P_B^{sm}(c,d)=0$, if (c,d)=(0,3),(1,3)
$P_A^{sm}(2,2)= p_A^{15s}$
$P_B^{sm}(2,2)= q_B^{15s}$

Let $P_A^{pm}(a,b:c,d)$ and $P_B^{pm}(a,b:c,d)$ represent the probabilities of player A winning a match at point and set score (a,b:c,d) for player A and player B serving respectively

$P_A^{pm}(a,b:c,d)=P_A^{25ps}(a,b)P_A^{sm}(c+1,d) +(1-P_A^{25ps}(a,b))P_B^{sm}(c,d+1)$, if $(c,d)\neq(2,2)$
$P_A^{pm}(a,b:c,d)=P_A^{15ps}(a,b)$, if (c,d)=(2,2)
$P_B^{pm}(a,b:c,d)= P_B^{25ps}(a,b)P_A^{sm}(c+1,d) +(1-P_B^{25ps}(a,b))P_B^{sm}(c,d+1)$, if $(c,d)\neq(2,2)$
$P_B^{pm}(a,b:c,d)= P_B^{15ps}(a,b)$, if (c,d)=(2,2)

	match	
p_A, p_B	A serving	B serving
0.30, 0.30	0.50	0.50
0.30, 0.29	0.56	0.56
0.30, 0.25	0.77	0.77
0.25, 0.25	0.50	0.50
0.25, 0.24	0.56	0.56
0.20, 0.20	0.50	0.50

Table: The probability of team A winning a match for different values of p_A and p_B from the start of the match

Mean number of points remaining in the set

Let $\mu(Y_A^{25ps}(a,b))$ and $\mu(Y_B^{25ps}(a,b))$ represent the mean number of points remaining in a first to 25 point set at point score (a,b) for player A and player B serving respectively

Recurrence formulas:

$\mu(Y_A^{25ps}(a,b)) = 1+p_A\,\mu(Y_A^{25ps}(a+1,b)) + q_A\,\mu(Y_B^{25ps}(a,b+1))$

$\mu(Y_B^{25ps}(a,b))=1+p_B\,\mu(Y_B^{25ps}(a,b+1))+q_B\,\mu(Y_A^{25ps}(a+1,b))$

Boundary values:

$\mu(Y_A^{25ps}(a,b)) = \mu(Y_B^{25ps}(a,b))=0$, if $a=25$, $0\leq b\leq 23$ or $b=25$, $0\leq a\leq 23$

$\mu(Y_A^{25ps}(24,24))= 2(1+p_Aq_A-q_Aq_B)/((1-q_Aq_B)^2-p_Aq_Ap_Bq_B)$

$\mu(Y_B^{25ps}(24,24))=2(1+p_Bq_B-q_Aq_B)/((1-q_Aq_B)^2-p_Aq_Ap_Bq_B)$

Let $\mu(Y_A^{15ps}(a,b))$ and $\mu(Y_B^{15ps}(a,b))$ represent the mean number of points remaining in a first to 15 point set at point score (a,b) for player A and player B serving respectively

Recurrence formulas:

$\mu(Y_A^{15ps}(a,b)) = 1+p_A\,\mu(Y_A^{15ps}(a+1,b)) + q_A\,\mu(Y_B^{15ps}(a,b+1))$

$\mu(Y_B^{15ps}(a,b))=1+p_B\,\mu(Y_B^{15ps}(a,b+1))+q_B\,\mu(Y_A^{15ps}(a+1,b))$

Boundary values:

$\mu(Y_A^{15ps}(a,b)) = \mu(Y_B^{15ps}(a,b))=0$, if $a=15$, $0\leq b\leq 13$ or $b=15$, $0\leq a\leq 13$

$\mu(Y_A^{15ps}(14,14))= 2(1+p_Aq_A-q_Aq_B)/((1-q_Aq_B)^2-p_Aq_Ap_Bq_B)$

$\mu(Y_B^{15ps}(14,14))=2(1+p_Bq_B-q_Aq_B)/((1-q_Aq_B)^2-p_Aq_Ap_Bq_B)$

	Mean		Standard deviation	
p_A, p_B	A serving	B serving	A serving	B serving
0.30, 0.30	47.0	47.0	4.7	4.7
0.30, 0.29	47.1	47.1	4.8	4.8
0.30, 0.25	47.2	47.0	5.0	5.0
0.25, 0.25	47.8	47.8	5.4	5.4
0.25, 0.24	47.9	47.8	5.5	5.5
0.20, 0.20	48.9	48.9	6.7	6.7

Table: The mean and standard deviation of the number of points in a first to 25 points set for different values of p_A and p_B

Variance of the number of points remaining in the set

Let $\sigma^2(Y_A^{25ps}(a,b))$ and $\sigma^2(Y_B^{25ps}(a,b))$ represent the variance of the number of points remaining in a first to 25 point set at point score (a,b) for player A and player B serving respectively

Recurrence formulas:

$\sigma^2(Y_A^{25ps}(a,b))=p_A\sigma^2(Y_A^{25ps}(a+1,b))+q_A\sigma^2(Y_B^{25ps}(a,b+1))+p_Aq_A(\mu(Y_A^{25ps}(a+1,b))-\mu(Y_B^{25ps}(a,b+1)))^2$

$\sigma^2(Y_B^{25ps}(a,b))=p_B\sigma^2(Y_B^{25ps}(a,b+1))+q_B\sigma^2(Y_A^{25ps}(a+1,b))+p_Bq_B(\mu(Y_B^{25ps}(a,b+1))-\mu(Y_A^{25ps}(a+1,b)))^2$

Boundary values:

$\mu(Y_A^{25ps}(a,b)) = \mu(Y_B^{25ps}(a,b))=0$, if $a=25$, $0\le b\le 23$ or $b=25$, $0\le a\le 23$

$\mu(Y_A^{25ps}(24,24))=4q_A(p_A+q_B+3p_Ap_Bq_B-2q_Aq_B^2-p_A^2q_A-p_Ap_Bq_Aq_B^2-p_Aq_A^2q_B^2+p_A^2p_Bq_Aq_B+q_A^2q_B^3)/D^2$

$\mu(Y_B^{25ps}(24,24))=4q_B(p_B+q_A+3p_Ap_Bq_A-2q_A^2q_B-p_B^2q_B-p_Ap_Bq_A^2q_B-p_Bq_A^2q_B^2+p_Ap_B^2q_Aq_B+q_A^3q_B^2)/D^2$

where $D=(1-q_Aq_B)^2-p_Aq_Ap_Bq_B$

Let $\sigma^2(Y_A^{15ps}(a,b))$ and $\sigma^2(Y_B^{15ps}(a,b))$ represent the variance of the number of points remaining in a first to 15 point set at point score (a,b) for player A and player B serving respectively

Recurrence formulas:

$\sigma^2(Y_A^{15ps}(a,b))=p_A\sigma^2(Y_A^{15ps}(a+1,b))+q_A\sigma^2(Y_B^{15ps}(a,b+1))+p_Aq_A(\mu(Y_A^{15ps}(a+1,b))-\mu(Y_B^{15ps}(a,b+1)))^2$

$\sigma^2(Y_B^{15ps}(a,b))=p_B\sigma^2(Y_B^{15ps}(a,b+1))+q_B\sigma^2(Y_A^{15ps}(a+1,b))+p_Bq_B(\mu(Y_B^{15ps}(a,b+1))-\mu(Y_A^{15ps}(a+1,b)))^2$

Boundary values:

$\mu(Y_A^{15ps}(a,b)) = \mu(Y_B^{15ps}(a,b))=0$, if $a=15$, $0\le b\le 13$ or $b=15$, $0\le a\le 13$

$\mu(Y_A^{25ps}(14,14))=4q_A(p_A+q_B+3p_Ap_Bq_B-2q_Aq_B^2-p_A^2q_A-p_Ap_Bq_Aq_B^2-p_Aq_A^2q_B^2+p_A^2p_Bq_Aq_B+q_A^2q_B^3)/D^2$

$\mu(Y_B^{25ps}(14,14))=4q_B(p_B+q_A+3p_Ap_Bq_A-2q_A^2q_B-p_B^2q_B-p_Ap_Bq_A^2q_B-p_Bq_A^2q_B^2+p_Ap_B^2q_Aq_B+q_A^3q_B^2)/D^2$

where $D=(1-q_Aq_B)^2-p_Aq_Ap_Bq_B$

Mean number of sets remaining in the match

Let $\mu(Y_A^{sm}(a,b))$ $\mu(Y_B^{sm}(a,b))$ and represent the mean number of sets remaining in a match at set score (c,d) for player A and player B serving respectively

Recurrence formulas:

$\mu(Y_A^{sm}(a,b))=1+p_A^{25s}\mu(Y_B^{sm}(c+1,d))+q_A^{25s}\mu(Y_B^{sm}(c,d+1))$

$\mu(Y_B^{sm}(a,b))=1+q_B^{25s}\mu(Y_A^{sm}(c+1,d))+p_B^{25s}\mu(Y_A^{sm}(c,d+1))$

Boundary values:

$\mu(Y_A^{sm}(a,b))=\mu(Y_B^{sm}(a,b))=0$, if $(c,d)=(3,0),(3,1),(0,3),(1,3)$

$\mu(Y_A^{sm}(2,2))=\mu(Y_B^{sm}(2,2))=1$

Let $\mu(Y_A^{sm}(a,b:c,d))$ and $\mu(Y_B^{sm}(a,b:c,d))$ represent the mean number of sets remaining in a match at point and set score (a,b:c,d) for player A and player B serving respectively

$$\mu(Y_A^{sm}(a,b:c,d))=1+P_A^{25ps}(a,b)\mu(Y_B^{sm}(c+1,d))+(1-P_A^{25ps}(a,b))\mu(Y_B^{sm}(c,d+1))$$
$$\mu(Y_B^{sm}(a,b:c,d))=1+P_B^{25ps}(a,b)\mu(Y_A^{sm}(c+1,d))+(1-P_B^{25ps}(a,b))\mu(Y_A^{sm}(c,d+1))$$

Variance of the number of sets remaining in the match

Let $\sigma^2(Y_A^{sm}(a,b))$ and $\sigma^2(Y_B^{sm}(a,b))$ represent the variance of the number of sets remaining in a match at set score (c,d) for player A and player B serving respectively

Recurrence formulas:
$$\sigma^2(Y_A^{sm}(a,b))=p_A^{25s}\sigma^2(Y_B^{sm}(c+1,d))+q_A^{25s}\sigma^2(Y_B^{sm}(c,d+1))+p_A^{25s}q_A^{25s}(\mu(Y_B^{sm}(c+1,d))-\mu(Y_B^{sm}(c,d+1)))^2$$
$$\sigma^2(Y_B^{sm}(a,b))=q_B^{25s}\sigma^2(Y_A^{sm}(c+1,d))+p_B^{25s}\sigma^2(Y_A^{sm}(c,d+1))+q_B^{25s}p_B^{25s}(\mu(Y_A^{sm}(c+1,d))-\mu(Y_A^{sm}(c,d+1)))^2$$

Boundary values:
$$\sigma^2(Y_A^{sm}(a,b))=\sigma^2(Y_B^{sm}(a,b))=0, \text{ if } (c,d)=(3,0),(3,1),(0,3),(1,3)$$
$$\sigma^2(Y_A^{sm}(2,2))=\sigma^2(Y_B^{sm}(2,2))=0$$

Let $\sigma^2(Y_A^{sm}(a,b:c,d))$ and $\sigma^2(Y_B^{sm}(a,b:c,d))$ represent the variance of the number of sets remaining in a match at point and set score (a,b:c,d) for player A and player B serving respectively

$$\sigma^2(Y_A^{sm}(a,b:c,d))=P_A^{25ps}(a,b)\sigma^2(Y_B^{sm}(c+1,d))+(1-P_A^{25ps}(a,b))\sigma^2(Y_B^{sm}(c,d+1))+P_A^{25ps}(a,b)(1-P_A^{25ps}(a,b))(\mu(Y_B^{sm}(c+1,d))-\mu(Y_B^{sm}(c,d+1)))^2$$
$$\sigma^2(Y_B^{sm}(a,b:c,d))=P_B^{25ps}(a,b)\sigma^2(Y_A^{sm}(c+1,d))+(1-P_B^{25ps}(a,b))\sigma^2(Y_A^{sm}(c,d+1))+P_B^{25ps}(a,b)(1-P_B^{25ps}(a,b))(\mu(Y_A^{sm}(c+1,d))-\mu(Y_A^{sm}(c,d+1)))^2$$

	Mean		Standard deviation	
p_A, p_B	A serving	B serving	A serving	B serving
0.30, 0.30	4.1	4.1	0.8	0.8
0.30, 0.29	4.1	4.1	0.8	0.8
0.30, 0.25	4.0	4.0	0.8	0.8
0.25, 0.25	4.1	4.1	0.8	0.8
0.25, 0.24	4.1	4.1	0.8	0.8
0.20, 0.20	4.1	4.1	0.8	0.8

Table: The mean and standard deviation of the number sets in a match for different values of p_A and p_B