A recursive approach to modelling the amount of time played in a tennis match

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Introduction

Several papers have been written by the author to model outcomes in a tennis match using recursive formulas. Barnett and Clarke (2005) calculate the chances of winning a standard game conditional on the point score. Barnett (2011) calculates the chances of winning a standard and tiebreak game conditional on the point score, a tiebreak set conditional on the point and game score and a match conditional on the point, game and set score. Barnett (2013) calculates the distribution and the parameters of distribution (mean and variance) of the number of points played in a standard game. Barnett (2014) calculates the parameters of distribution (mean, variance, coefficients of skewness and kurtosis) of the number of points played in a game. This paper is an extension to the above papers by using recursive formulas to obtain the parameters of distribution (mean, variance, coefficients of skewness and kurtosis) of both the number of points played in an advantage set conditional on the point and game score and the number of points played in a match conditional on the point, game and set score. Based on these calculations, the parameters of distribution of the time played in a match conditional on the point, game and set score are obtained. An example is given to illustrate why the standard deviation can be insufficient information for measuring risk.

Probability of winning a game

Let p_A represent the probability of player A winning a point on serve
Let p_B represent the probability of player B winning a point on serve
Let q_A=1-p_A and q_B=1-p_B

Let $P^{pg}(a,b|c_A,w_A)$ represent the probability of player A winning a game at point score (a,b) for player A serving

Recurrence formula:
$P^{pg}(a,b|c_A,w_A)=p_AP^{pg}(a+1,b|c_A,w_A)+q_AP^{pg}(a,b+1|c_A,w_A)$

Boundary Values:
$P^{pg}(a,b|c_A,w_A)=1$, if a=4 and b≤2
$P^{pg}(a,b|c_A,w_A)=0$, if b=4 and a≤2
$P^{pg}(3,3|c_A,w_A)= p_A^2/(p_A^2+q_A^2)$

The boundary values and recursion formula can be entered on a simple spreadsheet (such as Excel). By using relative and absolute referencing in Excel, the recursion formula can be copied and pasted into other cells. Thus modelling a tennis match using this recursion technique is simple to implement for obtaining numerical results.

Table 1 represents the conditional probabilities of player A winning the game from various score lines for p_A=0.6. It indicates that a player with a 60% chance of winning a point has a 73.6% chance of winning the game. Note that since advantage server is logically equivalent to 40-30, and advantage receiver is logically equivalent to 30-40, the required statistics can be found from these cells. Also worth noting is that the chances of winning from deuce and 30-30 are the same.

				B score		
		0	15	30	40	game
	0	0.736	0.576	0.369	0.150	0
	15	0.842	0.714	0.515	0.249	0
A score	30	0.927	0.847	0.692	0.415	0
	40	0.980	0.951	0.877	0.692	
	game	1	1	1		

Table 1: The conditional probabilities of player A winning the game on serve from various score lines for p_A=0.6

Let $P^{pg}(a,b|c_B,w_B)$ represent the probability of player B winning a game at point score (a,b) for player B serving.

Similar recurrence formulas and boundary values can be obtained for $P^{pg}(a,b|c_B,w_B)$.

It follows that $P^{pg}(a,b|c_A,l_A)=1-P^{pg}(a,b|c_A,w_A)$ and $P^{pg}(a,b|c_B,l_B)=1-P^{pg}(a,b|c_B,w_B)$, where l_A and l_B represent player A and player B losing the game respectively.

Number of points in a game

Let $M_{Ypg(a,b|cA,wA)}(t)$ represent the moment generating function for the number of points remaining in a game at point score (a,b) for player A winning the game and serving.

Let $W_{Ypg(a,b|cA,wA)}(t)=P^{pg}(a,b|c_A,w_A)M_{Ypg(a,b|cA,wA)}(t)$ represent the weighted moment generating function for the number of points remaining in a game at point score (a,b) for player A winning the game and serving.

Without proof: $W_{Ypg(a,b|cA,wA)}(t)=p_Ae^tW_{Ypg(a+1,b|cA,wA)}(t)+q_Ae^tW_{Ypg(a,b+1|cA,wA)}(t)$

Let $w_n(Y^{pg}(a,b|c_A,w_A))=W^n{}_{Ypg(a,b|cA,wA)}(0)$ represent the weighted n^{th} moment of the number of points remaining in a game at point score (a,b) for player A winning the game and serving.

Recurrence formulas:

$w_1(Y^{pg}(a,b|c_A,w_A))$
$=p_Aw_1(Y^{pg}(a+1,b|c_A,w_A))+q_Aw_1(Y^{pg}(a,b+1|c_A,w_A))+p_AP^{pg}(a+1,b|c_A,w_A)+q_AP^{pg}(a,b+1|c_A,w_A)$

$w_2(Y^{pg}(a,b|c_A,w_A))$
$=p_Aw_2(Y^{pg}(a+1,b|c_A,w_A))+q_Aw_2(Y^{pg}(a,b+1|c_A,w_A))+2p_Aw_1(Y^{pg}(a+1,b|c_A,w_A))+2q_Aw_1(Y^{pg}(a,b+1|c_A,w_A))+$
$p_AP^{pg}(a+1,b|c_A,w_A)+q_AP^{pg}(a,b+1|c_A,w_A)$

$w_3(Y^{pg}(a,b|c_A,w_A))$
$=p_Aw_3(Y^{pg}(a+1,b|c_A,w_A))+q_Aw_3(Y^{pg}(a,b+1|c_A,w_A))+3p_Aw_2(Y^{pg}(a+1,b|c_A,w_A))+3q_Aw_2(Y^{pg}(a,b+1|c_A,w_A))+$
$3p_Aw_1(Y^{pg}(a+1,b|c_A,w_A))+3q_Aw_1(Y^{pg}(a,b+1|c_A,w_A))+p_AP^{pg}(a+1,b|c_A,w_A)+q_AP^{pg}(a,b+1|c_A,w_A)$

$w_4(Y^{pg}(a,b|c_A,w_A))$
$=p_A w_4(Y^{pg}(a+1,b|c_A,w_A))+q_A w_4(Y^{pg}(a,b+1|c_A,w_A))+4p_A w_3(Y^{pg}(a+1,b|c_A,w_A))+4q_A w_3(Y^{pg}(a,b+1|c_A,w_A))+6p_A w_2(Y^{pg}(a+1,b|c_A,w_A))+6q_A w_2(Y^{pg}(a,b+1|c_A,w_A))+4p_A w_1(Y^{pg}(a+1,b|c_A,w_A))+4q_A w_1(Y^{pg}(a,b+1|c_A,w_A))+p_A P^{pg}(a+1,b|c_A,w_A)+q_A P^{pg}(a,b+1|c_A,w_A)$

It can be shown that $W_{Ypg(3,3|cA,wA)}(t)=e^{2t}p_A^{\,2}/(1-2p_Aq_Ae^{2t})$.

Boundary Values:
$w_n(Y^{pg}(a,b|c_A,w_A))=0$, if a=4 and 0≤b≤2; b=4 and 0≤a≤2
$w_1(Y^{pg}(3,3|c_A,w_A))=2p_A^{\,2}/(2p_A^{\,2}-2p_A+1)^2$
$w_2(Y^{pg}(3,3|c_A,w_A))=4p_A^{\,2}(1-2p_A^{\,2}+2p_A)/(2p_A^{\,2}-2p_A+1)^3$
$w_3(Y^{pg}(3,3|c_A,w_A))=8p_A^{\,2}(4p_A^{\,4}-8p_A^{\,3}-4p_A^{\,2}+8p_A+1)/(2p_A^{\,2}-2p_A+1)^4$
$w_4(Y^{pg}(3,3|c_A,w_A))=16p_A^{\,2}(1-2p_A^{\,2}+2p_A)(4p_A^{\,4}-8p_A^{\,3}-16p_A^{\,2}+20p_A+1)/(2p_A^{\,2}-2p_A+1)^5$

Table 2 represents the weighted first moment of the number of points remaining in a game at point score (a,b) given player A is serving and wins the game for p_A=0.6

				B score		
		0	15	30	40	game
	0	4.7	3.7	2.4	1.0	0
	15	4.1	3.6	2.7	1.5	0
A score	30	3.1	2.9	2.7	2.0	0
	40	1.7	1.7	1.9	2.7	
	game	0	0	0		

Table 2: The weighted first moment of the number of points remaining in a game at point score (a,b) given player A is serving and wins the game for p_A=0.6

Let $w_n(Y^{pg}(a,b|c_A,l_A))$ represent the weighted n^{th} moment of the number of points remaining in a game at point score (a,b) for player A losing the game and serving.

Similar recurrence formulas and boundary conditions can be obtained for $w_n(Y^{pg}(a,b|c_A,l_A))$, where $w_n(Y^{pg}(a,b|c_A,l_A))=W^n_{Ypg(a,b|cA,lA)}(0)$.

Let $w_n(Y^{pg}(a,b|c_A))$ represent the weighted n^{th} moment of the number of points remaining in a game at point score (a,b) for player A serving.

It can be shown that $w_n(Y^{pg}(a,b|c_A))=w_n(Y^{pg}(a,b|c_A,w_A))+w_n(Y^{pg}(a,b|c_A,l_A))$

Let $\mu(Y^{pg}(a,b|c_A))$, $\sigma^2(Y^{pg}(a,b|c_A))$, $\gamma_1(Y^{pg}(a,b|c_A))$ and $\gamma_2(Y^{pg}(a,b|c_A))$ represent the mean, variance, coefficient of skewness and coefficient of excess kurtosis of the number of points remaining in a game at point score (a,b) for player A serving.

Let $m_n(Y^{pg}(a,b|c_A))$ represent the n^{th} moment of the number of points remaining in a game at point score (a,b) for player A serving.

It can be shown that $m_n(Y^{pg}(a,b|c_A))=w_n(Y^{pg}(a,b|c_A))$, and the standard results follow for obtaining parameters of a distribution from moments:

$\mu(Y^{pg}(a,b|c_A))=m_1(Y^{pg}(a,b|c_A))$
$\sigma^2(Y^{pg}(a,b|c_A))=m_2(Y^{pg}(a,b|c_A))-m_1(Y^{pg}(a,b|c_A))^2$
$\gamma_1(Y^{pg}(a,b|c_A))=(m_3(Y^{pg}(a,b|c_A))-3m_2(Y^{pg}(a,b|c_A))m_1(Y^{pg}(a,b|c_A))+2m_1(Y^{pg}(a,b|c_A))^3)/(\sigma^2(Y^{pg}(a,b|c_A)))^{3/2}$
$\gamma_2(Y^{pg}(a,b|c_A))=(m_4(Y^{pg}(a,b|c_A))-4m_3(Y^{pg}(a,b|c_A))m_1(Y^{pg}(a,b|c_A))-3m_2(Y^{pg}(a,b|c_A))^2+12m_2(Y^{pg}(a,b|c_A))$
$m_1(Y^{pg}(a,b|c_A))^2-6m_1(Y^{pg}(a,b|c_A))^4)/(\sigma^2(Y^{pg}(a,b|c_A)))^2$

Table 3 represents the mean number of points remaining in a game at point score (a,b) given player A is serving for p_A=0.6.

			B score		
		0	15	30	40
	0	6.5	6.0	4.8	2.8
A score	15	5.2	5.0	4.5	3.0
	30	3.6	3.7	3.8	3.3
	40	1.8	2.0	2.5	3.8

Table 3: The mean number of points remaining in a game at point score (a,b) given player A is serving for p_A=0.6

Similar derivations can be obtained for player B serving where $w_n(Y^{pg}(a,b|c_B,w_A))$ and $w_n(Y^{pg}(a,b|c_B,l_A))$ represent the weighted n^{th} moment of the number of points remaining in a game at point score (a,b) for player B serving, and winning and losing the game respectively.

Probability of winning an advantage set

Let $P^{gs}(c,d|s_A,w_A,n_A)$ represent the probability of player A winning an advantage set at game score (c,d) given player A served first (s_A) and is serving first in the next set if required (n_A).

Recurrence Formulas:
$P^{gs}(c,d|s_A,w_A,n_A)=P^{pg}(0,0|c_A,w_A)P^{gs}(c+1,d|s_A,w_A,n_A)+P^{pg}(0,0|c_A,l_A)P^{gs}(c,d+1|s_A,w_A,n_A)$, if (c+d)mod2=0
$P^{gs}(c,d|s_A,w_A,n_A)= P^{pg}(0,0|c_B,l_B)P^{gs}(c+1,d|s_A,w_A,n_A)+P^{pg}(0,0|c_B,w_B)P^{gs}(c,d+1|s_A,w_A,n_A)$, if (c+d)mod 2=1

Boundary Values:
$P^{gs}(c,d|s_A,w_A,n_A)=1$, if (6, 0); (6, 2); (6, 4); (7,5)
$P^{gs}(c,d|s_A,w_A,n_A)=0$, if d=6 and $0 \leq c \leq 4$; (6,1); (6,3); (5,7)
$P^{gs}(6,6|s_A,w_A,n_A) = P^{pg}(0,0|c_A,w_A)P^{pg}(0,0|c_B,l_B)/(1-P^{pg}(0,0|c_A,w_A)P^{pg}(0,0|c_B,w_B)-P^{pg}(0,0|c_A,l_A)P^{pg}(0,0|c_B,l_B))$

Table 4 represents the probability of player A winning an advantage set at game score (c,d) given player A served first and is serving first in the next set if required for p_A=0.62 and p_B=0.60.

				B score					
		0	1	2	3	4	5	6	7
	0	0.310	0.242	0.210	0.092	0.053	0.006	0	
	1	0.330	0.329	0.245	0.202	0.066	0.023	0	
A score	2	0.333	0.353	0.362	0.242	0.185	0.030	0	
	3	0.328	0.330	0.397	0.399	0.230	0.114	0	
	4	0.323	0.310	0.391	0.448	0.554	0.146	0	
	5	0.326	0.084	0.375	0.151	0.672	0.554	0.146	0
	6	1	0	1	0	1	0.672	0.554	
	7						1		

Table 4: The probability of player A winning an advantage set at game score (c,d) given player A served first and is serving first in the next set to be played for p_A=0.62 and p_B=0.60.

Let $P^{ps}(a,b:c,d|s_A,w_A,n_A)$ represent the probability of player A winning an advantage set at point and game score (a,b:c,d) given player A served first and is serving first in the next set if required.

It follows that:

$P^{ps}(a,b:c,d|s_A,w_A,n_A)=P^{pg}(a,b|c_A,w_A)P^{gs}(c+1,d|s_A,w_A,n_A)+P^{pg}(a,b|c_A,l_A)P^{gs}(c,d+1|s_A,w_A,n_A)$, if (c+d)mod 2 = 0

$P^{ps}(a,b:c,d|s_A,w_A,n_A)=P^{pg}(a,b|c_B,l_B)P^{gs}(c+1,d|s_A,w_A,n_A)+P^{pg}(a,b|c_B,w_B)P^{gs}(c,d+1|s_A,w_A,n_A)$, if (c+d)mod 2 =1

Similar derivations can be obtained for $P^{ps}(a,b:c,d|s_A,w_A,n_B)$, $P^{ps}(a,b:c,d|s_A,l_A,n_A)$, $P^{ps}(a,b:c,d|s_A,l_A,n_B)$, $P^{ps}(a,b:c,d|s_B,w_B,n_A)$, $P^{ps}(a,b:c,d|s_B,w_B,n_B)$, $P^{ps}(a,b:c,d|s_B,l_B,n_A)$ and $P^{ps}(a,b:c,d|s_B,l_B,n_B)$.

Number of points in an advantage set

Let $W_{Yps(a,b:c,d|sA,wA,nA)}(t)$ represent the weighted moment generating function of the number of points remaining in an advantage set at point and game score (a,b:c,d) given player A served first, wins the set and is serving first in the next set if required.

Without proof:

$W_{Yps(0,0:c,d|sA,wA,nA)}(t)$

$=W_{Ypg(a,b|cA,wA)}(t)W_{Yps(0,0:c+1,d|sA,wA,nA)}(t)+W_{Ypg(a,b|cA,lA)}(t)W_{Yps(0,0:c,d+1|sA,wA,nA)}(t)$, for (c+d) mod 2=0

$W_{Yps(0,0:c,d|sA,wA,nA)}(t)$

$=W_{Ypg(a,b|cB,lB)(t)WYps(0,0:c+1,d|sA,wA,nA)}(t)+W_{Ypg(a,b|cB,wB)}(t)W_{Yps(0,0:c,d+1|sA,wA,nA)}(t)$, for (c+d) mod 2=1

Let $w_n(Y^{ps}(a,b:c,d|s_A,w_A,n_A))$ represent the weighted n^{th} moment of the number of points remaining in an advantage set at point and game score (a,b:c,d) given player A served first (s_A), wins the set (w_A) and is serving first in the next set if required (n_A).

Recurrence formulas:

$w_1(Y^{ps}(0,0:c,d|s_A,w_A,n_A))$

$=P^{pg}(0,0|c_A,w_A)w_1(Y^{ps}(0,0:c+1,d|s_A,w_A,n_A))+P^{pg}(0,0|c_A,l_A)w_1(Y^{ps}(0,0:c,d+1|s_A,w_A,n_A))$ +

$w_1(Y^{pg}(0,0|c_A,w_A))P^{gs}(c+1,d|s_A,w_A,n_A)+w_1(Y^{pg}(0,0|c_A,l_A))P^{gs}(c,d+1|s_A,w_A,n_A)$, if (c+d)mod 2=0

$w_2(Y^{ps}(0,0:c,d|s_A,w_A,n_A))$

$= P^{pg}(0,0|c_A,w_A)w_2(Y^{ps}(0,0:c+1,d|s_A,w_A,n_A)) + P^{pg}(0,0|c_A,l_A)w_2(Y^{ps}(0,0:c,d+1|s_A,w_A,n_A))$

$+ 2w_1(Y^{pg}(0,0|c_A,w_A))w_1(Y^{ps}(0,0:c+1,d|s_A,w_A,n_A)) + 2w_1(Y^{pg}(0,0|c_A,l_A))w_1(Y^{ps}(0,0:c,d+1|s_A,w_A,n_A))$

$+ w_2(Y^{pg}(0,0|c_A,w_A))P^{gs}(c+1,d|s_A,w_A,n_A) + w_2(Y^{pg}(0,0|c_A,l_A))P^{gs}(c,d+1|s_A,w_A,n_A)$, if (c+d)mod 2=0

$w_3(Y^{ps}(0,0:c,d|s_A,w_A,n_A))$

$= P^{pg}(0,0|c_A,w_A)w_3(Y^{ps}(0,0:c+1,d|s_A,w_A,n_A)) + P^{pg}(0,0|c_A,l_A)w_3(Y^{ps}(0,0:c,d+1|s_A,w_A,n_A))$

$+ 3w_1(Y^{pg}(0,0|c_A,w_A))w_2(Y^{ps}(0,0:c+1,d|s_A,w_A,n_A)) + 3w_1(Y^{pg}(0,0|c_A,l_A))w_2(Y^{ps}(0,0:c,d+1|s_A,w_A,n_A))$

$+3w_2(Y^{pg}(0,0|c_A,w_A))w_1(Y^{ps}(0,0:c+1,d|s_A,w_A,n_A)) + 3w_2(Y^{pg}(0,0|c_A,l_A))w_1(Y^{ps}(0,0:c,d+1|s_A,w_A,n_A))$

$+w_3(Y^{pg}(0,0|c_A,w_A))P^{gs}(c+1,d|s_A,w_A,n_A) + w_3(Y^{pg}(0,0|c_A,l_A))P^{gs}(c,d+1|s_A,w_A,n_A)$, if (c+d)mod 2=0

$w_4(Y^{ps}(0,0{:}c,d|s_A,w_A,n_A))$
$= P^{pg}(0,0|c_A,w_A)w_4(Y^{ps}(0,0{:}c+1,d|s_A,w_A,n_A))+P^{pg}(0,0|c_A,l_A)w_4(Y^{ps}(0,0{:}c,d+1|s_A,w_A,n_A))$
$+ 4w_1(Y^{pg}(0,0|c_A,w_A))w_3(Y^{ps}(0,0{:}c+1,d|s_A,w_A,n_A))+4w_1(Y^{pg}(0,0|c_A,l_A))w_3(Y^{ps}(0,0{:}c,d+1|s_A,w_A,n_A))$
$+ 6w_2(Y^{pg}(0,0|c_A,w_A))w_2(Y^{ps}(0,0{:}c+1,d|s_A,w_A,n_A))+6w_2(Y^{pg}(0,0|c_A,l_A))w_2(Y^{ps}(0,0{:}c,d+1|s_A,w_A,n_A))$
$+ 4w_3(Y^{pg}(0,0|c_A,w_A))w_1(Y^{ps}(0,0{:}c+1,d|s_A,w_A,n_A))+4w_3(Y^{pg}(0,0|c_A,l_A))w_1(Y^{ps}(0,0{:}c,d+1|s_A,w_A,n_A))$
$+ w_4(Y^{pg}(0,0|c_A,w_A))P^{gs}(c+1,d|s_A,w_A,n_A)+w_4(Y^{pg}(0,0|c_A,l_A))P^{gs}(c,d+1|s_A,w_A,n_A)$, if (c+d)mod 2=0

$w_1(Y^{ps}(0,0{:}c,d|s_A,w_A,n_A))$
$= P^{pg}(0,0|c_B,l_B)w_1(Y^{ps}(0,0{:}c+1,d|s_A,w_A,n_A))+P^{pg}(0,0|c_B,w_B)w_1(Y^{ps}(0,0{:}c,d+1|s_A,w_A,n_A))$
$+ w_1(Y^{pg}(0,0|c_B,l_B))P^{gs}(c+1,d|s_A,w_A,n_A)+w_1(Y^{pg}(0,0|c_B,w_B))P^{gs}(c,d+1|s_A,w_A,n_A)$, if (c+d)mod 2=1

Similar recurrence formulas can be obtained for $w_2(Y^{ps}(0,0{:}c,d|s_A,w_A,n_A))$, $w_3(Y^{ps}(0,0{:}c,d|s_A,w_A,n_A))$ and $w_4(Y^{ps}(0,0{:}c,d|s_A,w_A,n_A))$, if (c+d)mod 2=1

Without proof:
$W_{Yps(0,0:6,6|sA,wA,nA)}(t)$
$=W_{Ypg(0,0|cA,wA)}(t)W_{Ypg(0,0|cB,lB)}(t)/(1-W_{Ypg(0,0|cA,wA)}(t)W_{Ypg(0,0|cB,wB)}(t)-W_{Ypg(0,0|cA,lA)}(t)W_{Ypg(0,0|cB,lB)}(t))$

Boundary values:
$w_n(Y^{ps}(0,0{:}c,d|s_A,w_A,n_A))=0$, if c=6 and 0≤d≤4; d=6 and 0≤c≤4; (7,5); (5,7)
$w_1(Y^{ps}(0,0{:}6,6|s_A,w_A,n_A))=(X4*Y3+X3*Y4)/(1-W3)$
$w_2(Y^{ps}(0,0{:}6,6|s_A,w_A,n_A))=(X5*Y3+2*X4*Y4+X3*Y5)/(1-W3)$
$w_3(Y^{ps}(0,0{:}6,6|s_A,w_A,n_A))=(X6*Y3+3*X5*Y4+3*X4*Y5+X3*Y6)/(1-W3)$
$w_4(Y^{ps}(0,0{:}6,6|s_A,w_A,n_A))=(X7*Y3+4*X6*Y4+6*X5*Y5+4*X4*Y6+X3*Y7)/(1-W3)$

where:

X3=1 X4=(W4*X3)/(1-W3) X5=(W5*X3+2*W4*X4)/(1-W3) X6=(W6*X3+3*W5*X4+3*W4*X5)/(1-W3) X7=(W7*X3+4*W6*X4+6*W5*X5+4*W4*X6)/(1-W3)	Y3=F2*F5 Y4=F6*F5+F2*F9 Y5=F10*F5+2*F6*F9+F2*F13 Y6=F14*F5+3*F10*F9+3*F6*F13+F2*F17 Y7=F18*F5+4*F14*F9+6*F10*F13+4*F6*F17+F2*F21

W3=F2*F4+F3*F5 W4=F6*F4+F2*F8+F7*F5+F9*F3 W5=F10*F4+2*F6*F8+F2*F12+F11*F5+2*F7*F9+F3*F13 W6=F14*F4+3*F10*F8+3*F6*F12+F2*F16+F15*F5+3*F11*F9+3*F7*F13+F3*F17 W7=F18*F4+4*F14*F8+6*F10*F12+4*F6*F16+F2*F20+F19*F5+4*F15*F9+6*F11*F13+4*F7*F17+F3*F21

$F2=P^{pg}(0,0\|c_A,w_A)$	$F6=w_1(Y^{pg}(0,0\|c_A,w_A))$	$F10=w_2(Y^{pg}(0,0\|c_A,w_A))$	$F14=w_3(Y^{pg}(0,0\|c_A,w_A))$	$F18=w_4(Y^{pg}(0,0\|c_A,w_A))$
$F3= P^{pg}(0,0\|c_A,l_A)$	$F7=w_1(Y^{pg}(0,0\|c_A,l_A))$	$F11=w_2(Y^{pg}(0,0\|c_A,l_A))$	$F15=w_3(Y^{pg}(0,0\|c_A,l_A))$	$F19=w_4(Y^{pg}(0,0\|c_A,l_A))$,
$F4= P^{pg}(0,0\|c_B,w_B)$	$F8=w_1(Y^{pg}(0,0\|c_B,w_B))$	$F12=w_2(Y^{pg}(0,0\|c_B,w_B))$	$F16=w_3(Y^{pg}(0,0\|c_B,w_B))$	$F20=w_4(Y^{pg}(0,0\|c_B,w_B))$
$F5= P^{pg}(0,0\|c_B,l_B)$	$F9=w_1(Y^{pg}(0,0\|c_B,l_B))$	$F13=w_2(Y^{pg}(0,0\|c_B,l_B))$	$F17=w_3(Y^{pg}(0,0\|c_B,l_B))$	$F21=w_4(Y^{pg}(0,0\|c_B,l_B))$

Table 5 represents the weighted first moment of the number of points remaining in an advantage set at game score (c,d) for player A serving first, wins the set and serving first in the next set if required given p_A=0.62 and p_B=0.60

				B score					
		0	1	2	3	4	5	6	7
	0	24.6	19.1	15.7	6.7	3.6	0.4	0	
	1	23.6	22.5	16.6	12.9	4.1	1.4	0	
A score	2	18.7	21.5	20.7	13.8	9.8	1.6	0	
	3	15.1	15.1	19.7	19.2	10.6	5.4	0	
	4	7.4	11.0	11.5	18.4	19.2	6.1	0	
	5	3.7	2.0	6.3	5.2	18.6	19.2	6.1	0
	6	0	0	0	0	0	18.6	19.2	
	7						0		

Table 5: The weighted first moment of the number of points remaining in an advantage set at game score (c,d) for player A serving first, wins the set and serving first in the next set if required given p_A=0.62 and p_B=0.60

It follows that:

$w_1(Y^{ps}(a,b{:}c,d|s_A,w_A,n_A))$

$=P^{pg}(a,b|c_A,w_A)w_1(Y^{ps}(0,0{:}c+1,d|s_A,w_A,n_A))+P^{pg}(a,b|c_A,l_A)w_1(Y^{ps}(0,0{:}c,d+1|s_A,w_A,n_A))$

$+ w_1(Y^{pg}(a,b|c_A,w_A))P^{gs}(c+1,d|s_A,w_A,n_A)+w_1(Y^{pg}(a,b|c_A,l_A))P^{gs}(c,d+1|s_A,w_A,n_A)$, if (c+d) mod 2=0

Similar formulas can be obtained for $w_2(Y^{ps}(a,b{:}c,d|s_A,w_A,n_A))$, $w_3(Y^{ps}(a,b{:}c,d|s_A,w_A,n_A))$ and $w_4(Y^{ps}(a,b{:}c,d|s_A,w_A,n_A))$, if (c+d)mod 2=0

It follows that:

$w_1(Y^{ps}(a,b{:}c,d|s_A,w_A,n_A))$

$=P^{pg}(a,b|c_B,l_B)w_1(Y^{ps}(0,0{:}c+1,d|s_A,w_A,n_A)) + P^{pg}(a,b|c_B,w_B)w_1(Y^{ps}(0,0{:}c,d+1|s_A,w_A,n_A))$

$+ w_1(Y^{pg}(a,b|c_B,l_B))P^{gs}(c+1,d|s_A,w_A,n_A) + w_1(Y^{pg}(a,b|c_B,w_B))P^{gs}(c,d+1|s_A,w_A,n_A)$, if (c+d) mod 2=1

Similar formulas can be obtained for $w_2(Y^{ps}(a,b{:}c,d|s_A,w_A,n_A))$, $w_3(Y^{ps}(a,b{:}c,d|s_A,w_A,n_A))$ and $w_4(Y^{ps}(a,b{:}c,d|s_A,w_A,n_A))$, if (c+d)mod 2=1

Similar derivations can be obtained for $w_n(Y^{ps}(a,b{:}c,d|s_A,w_A,n_B))$, $w_n(Y^{ps}(a,b{:}c,d|s_A,l_A,n_A))$, $w_n(Y^{ps}(a,b{:}c,d|s_A,l_A,n_B))$, $w_n(Y^{ps}(a,b{:}c,d|s_B,w_B,n_A))$, $w_n(Y^{ps}(a,b{:}c,d|s_B,w_B,n_B))$, $w_n(Y^{ps}(a,b{:}c,d|s_B,l_B,n_A))$ and $w_n(Y^{ps}(a,b{:}c,d|s_B,l_B,n_B))$.

Let $w_n(Y^{ps}(a,b{:}c,d|s_A))$ represent the weighted n^{th} moment of the number of points remaining in an advantage set at point and game score (a,b:c,d) given player A served first (s_A)).

It can be shown that:

$w_n(Y^{ps}(a,b{:}c,d|s_A))$

$=w_n(Y^{ps}(a,b{:}c,d|s_A,w_A,n_A))+w_n(Y^{ps}(a,b{:}c,d|s_A,w_A,n_B))+w_n(Y^{ps}(a,b{:}c,d|s_A,l_A,n_A))+w_n(Y^{ps}(a,b{:}c,d|s_A,l_A,n_B))$

Let $m_n(Y^{pg}(a,b{:}c,d|s_A))$ represent the n^{th} moment of the number of points remaining in an advantage set at point and game score (a,b:c,d) for player A serving first in the set.

It can be shown that $m_n(Y^{pg}(a,b{:}c,d|s_A))=w_n(Y^{pg}(a,b{:}c,d|s_A))$, and thus standard results apply for obtaining parameters of a distribution from moments.

Table 6 represents the mean number of points remaining in an advantage set at game score (c,d) for player A serving first given p_A=0.62 and p_B=0.60.

				B score				
		0	1	2	3	4	5	6
	0	69.4	62.7	55.2	39.3	28.9	10.5	
	1	63.2	59.0	51.6	43.6	26.0	15.2	
A score	2	50.2	52.9	49.2	40.5	31.7	11.4	
	3	41.2	38.7	43.5	40.6	29.3	18.5	
	4	23.7	29.1	27.1	35.6	34.8	15.7	
	5	13.9	10.1	16.5	13.6	32.1	34.8	15.7
	6						32.1	34.8

Table 6: The mean number of points remaining in an advantage set at various score lines for player A serving first given p_A=0.62 and p_B=0.60

A similar derivation can be obtained for a tiebreak set.

Probability of winning a match

Let $P^{sm5}(e,f|c_A,w_A)$ represent the probability of player A winning a best-of-5 final advantage set match at set score (e,f) given player A wins the match and is currently serving.

Recurrence Formula:

$P^{sm5}(e,f|c_A,w_A) = P^{gsT}(0,0|s_A,w_A,n_A)P^{sm5}(e+1,f|c_A,w_A)+P^{gsT}(0,0|s_A,l_A,n_A)P^{sm5}(e,f+1|c_A,w_A)+$
$P^{gsT}(0,0|s_A,w_A,n_B)P^{sm5}(e+1,f|c_B,w_A)+P^{gsT}(0,0|s_A,l_A,n_B)P^{sm5}(e,f+1|c_B,w_A)$

Boundary Values:

$P^{sm5}(e,f|c_A,w_A)=1$, if (3,0); (3,1)
$P^{sm5}(e,f|c_A,w_A)=0$, if (0,3); (1,3)
$P^{sm5}(2,2|c_A,w_A)=P^{gs}(0,0|s_A,w_A)$

Table 7 represents the probability of player A winning a best-of-5 final advantage set match at set score (e,f) given player A wins the match and is currently serving, where p_A=0.62 and p_B=0.60

			B score		
		0	1	2	3
	0	0.627	0.422	0.184	0
A score	1	0.783	0.603	0.325	0
	2	0.920	0.815	0.572	
	3	1	1		

Table 7: The probability of player A winning a best-of-5 final advantage set match at set score (e,f) given player A wins the match and is currently serving, where p_A = 0.62 and p_B = 0.60

Number of points in a match

Let $W_{Ypm5(a,b:c,d:e,f|cA,wA)}(t)$ represent the weighted moment generating function of the number of points remaining in a best-of-5 final advantage set match at point, game and set score (a,b:c,d:e,f) given player A currently serving and wins the match.

Without proof:

$$W_{Ypm5}(0,0:0,0:e,f|c_A,w_A)(t)$$
$$=W_{YpsT}(0,0:0,0|s_A,w_A,n_A)(t)W_{Ypm5}(0,0:0,0:e+1,f|c_A,w_A)(t)+W_{YpsT}(0,0:0;0|s_A,l_A,n_A)(t)W_{Ypm5}(0,0:0,0:e,f+1|c_A,w_A)(t)$$
$$+W_{YpsT}(0,0:0,0|s_A,w_A,n_B)(t)W_{Ypm5}(0,0:0,0:e+1,f|c_B,w_A)(t)+W_{YpsT}(0,0:0,0|s_A,l_A,n_B)(t)W_{Ypm5}(0,0:0,0:e,f+1|c_B,w_A)(t),$$
if $0 \le e+f \le 4$

$$W_{Ypm5}(0,0:0,0:2,2|c_A,w_A)(t)= W_{Yps}(0,0:0,0|s_A,w_A)(t)$$

Let $w_n(Y^{pm5}(a,b:c,d:e,f|c_A,w_A))$ represent the weighted n^{th} moment of the number of points remaining in a best-of-5 final advantage set match at point, game and set score (a,b:c,d:e,f) given player A wins the match and is currently serving.

Recurrence formulas:

$$w_1(Y^{pm5}(0,0:0,0:e,f|c_A,w_A))$$
$$=P^{gsT}(0,0|s_A,w_A,n_A)w_1(Y^{pm5}(0,0:0,0:e+1,f|c_A,w_A))+P^{gsT}(0,0|s_A,l_A,n_A)w_1(Y^{pm5}(0,0:0,0:e,f+1|c_A,w_A))$$
$$+P^{gsT}(0,0|s_A,w_A,n_B)w_1(Y^{pm5}(0,0:0,0:e+1,f|c_B,w_A))+P^{gsT}(0,0|s_A,l_A,n_B)w_1(Y^{pm5}(0,0:0,0:e,f+1|c_B,w_A))$$
$$+w_1(Y^{psT}(0,0:0,0|s_A,w_A,n_A))P^{sm5}(e+1,f|c_A,w_A)+w_1(Y^{psT}(0,0:0,0|s_A,l_A,n_A))P^{sm5}(e,f+1|c_A,w_A)$$
$$+w_1(Y^{psT}(0,0:0,0|s_A,w_A,n_B))P^{sm5}(e+1,f|c_B,w_A)+w_1(Y^{psT}(0,0:0,0|s_A,l_A,n_B))P^{sm5}(e,f+1|c_B,w_A)$$

$$w_2(Y^{pm5}(0,0:0,0:e,f|c_A,w_A))$$
$$=P^{gsT}(0,0|s_A,w_A,n_A)w_2(Y^{pm5}(0,0:0,0:e+1,f|c_A,w_A))+P^{gsT}(0,0|s_A,l_A,n_A)w_2(Y^{pm5}(0,0:0,0:e,f+1|c_A,w_A))$$
$$+P^{gsT}(0,0|s_A,w_A,n_B)w_2(Y^{pm5}(0,0:0,0:e+1,f|c_B,w_A))+P^{gsT}(0,0|s_A,l_A,n_B)w_2(Y^{pm5}(0,0:0,0:e,f+1|c_B,w_A))$$
$$+2w_1(Y^{psT}(0,0:0,0|s_A,w_A,n_A))w_1(Y^{pm5}(0,0:0,0:e+1,f|c_A,w_A))+2w_1(Y^{psT}(0,0:0,0|s_A,l_A,n_A))w_1(Y^{pm5}(0,0:0,0: e,f+1|c_A,w_A))$$
$$+2w_1(Y^{psT}(0,0:0,0|s_A,w_A,n_B))w_1(Y^{pm5}(0,0:0,0:e+1,f|c_B,w_A))+2w_1(Y^{psT}(0,0:0,0|s_A,l_A,n_B))w_1(Y^{pm5}(0,0:0,0:e,f+1|c_B,w_A))$$
$$+w_2(Y^{psT}(0,0:0,0|s_A,w_A,n_A))P^{sm5}(e+1,f|c_A,w_A)+w_2(Y^{psT}(0,0:0,0|s_A,l_A,n_A))P^{sm5}(e,f+1|c_A,w_A)$$
$$+w_2(Y^{psT}(0,0:0,0|s_A,w_A,n_B))P^{sm5}(e+1,f|c_B,w_A)+w_2(Y^{psT}(0,0:0,0|s_A,l_A,n_B))P^{sm5}(e,f+1|c_B,w_A)$$

$$w_3(Y^{pm5}(0,0: 0,0:e,f|c_A,w_A))$$
$$=P^{gsT}(0,0|s_A,w_A,n_A)w_3(Y^{pm5}(0,0:0,0:e+1,f|c_A,w_A))+P^{gsT}(0,0|s_A,l_A,n_A)w_3(Y^{pm5}(0,0:0,0:e,f+1|c_A,w_A))$$
$$+P^{gsT} (0,0|s_A,w_A,n_B)w_3(Y^{pm5}(0,0:0,0:e+1,f|c_B,w_A))+P^{gsT}(0,0|s_A,l_A,n_B)w_3(Y^{pm5}(0,0:0,0:e,f+1|c_B,w_A))$$
$$+3w_1(Y^{psT}(0,0:0,0|s_A,w_A,n_A))w_2(Y^{pm5}(0,0:0,0:e+1,f|c_A,w_A))+3w_1(Y^{psT}(0,0:0,0|s_A,l_A,n_A))w_2(Y^{pm5}(0,0:0,0:e,f+1|c_A,w_A))$$
$$+3w_1(Y^{psT}(0,0:0,0|s_A,w_A,n_B))w_2(Y^{pm5}(0,0:0,0:e+1,f|c_B,w_A))+3w_1(Y^{psT}(0,0:0,0|s_A,l_A,n_B))w_2(Y^{pm5}(0,0:0,0:e,f+1|c_B,w_A))$$
$$+3w_2(Y^{psT}(0,0:0,0|s_A,w_A,n_A))w_3(Y^{pm5}(0,0:0,0:e+1,f|c_A,w_A))+3w_2(Y^{psT}(0,0:0,0|s_A,l_A,n_A))w_3(Y^{pm5}(0,0:0,0:e,f+1|c_A,w_A))$$
$$+3w_2(Y^{psT}(0,0:0,0|s_A,w_A,n_B))w_3(Y^{pm5}(0,0:0,0:e+1,f|c_B,w_A))+3w_2(Y^{psT}(0,0:0,0|s_A,l_A,n_B))w_3(Y^{pm5}(0,0:0,0:e,f+1|c_B,w_A))$$
$$+w_3(Y^{psT}(0,0:0,0|s_A,w_A,n_A))P^{sm5}(e+1,f|c_A,w_A)+w_3(Y^{psT}(0,0:0,0|s_A,l_A,n_A))P^{sm5}(e,f+1|c_A,w_A)$$
$$+w_3(Y^{psT}(0,0:0,0|s_A,w_A,n_B))P^{sm5}(e+1,f|c_B,w_A)+w_3(Y^{psT}(0,0:0,0|s_A,l_A,n_B))P^{sm5}(e,f+1|c_B,w_A)$$

$$w_4(Y^{pm5T}(0,0:0,0:e,f|c_A,w_A))$$
$$=P^{gsT}(0,0|s_A,w_A,n_A)w_4(Y^{pm5}(0,0:0,0:e+1,f|c_A,w_A))+P^{gsT}(0,0|s_A,l_A,n_A)w_4(Y^{pm5}(0,0:0,0:e,f+1|c_A,w_A))$$
$$+P^{gsT}(0,0|s_A,w_A,n_B)w_4(Y^{pm5}(0,0:0,0:e+1,f|c_B,w_A))+P^{gsT}(0,0|s_A,l_A,n_B)w_4(Y^{pm5}(0,0:0,0:e,f+1|c_B,w_A))$$
$$+4w_1(Y^{psT}(0,0:0,0|s_A,w_A,n_A))w_3(Y^{pm5}(0,0:0,0:e+1,f|c_A,w_A))+4w_1(Y^{psT}(0,0:0,0|s_A,l_A,n_A))w_3(Y^{pm5}(0,0:0,0:e,f+1|c_A,w_A))$$
$$+4w_1(Y^{psT}(0,0:0,0|s_A,w_A,n_B))w_3(Y^{pm5}(0,0:0,0:e+1,f|c_B,w_A))+4w_1(Y^{psT}(0,0:0,0|s_A,l_A,n_B))w_3(Y^{pm5}(0,0:0,0:e,f+1|c_B,w_A))$$
$$+6w_2(Y^{psT}(0,0:0,0|s_A,w_A,n_A))w_2(Y^{pm5}(0,0:0,0:e+1,f|c_A,w_A))+6w_2(Y^{psT}(0,0:0,0|s_A,l_A,n_A))w_2(Y^{pm5}(0,0:0,0:e,f+1|c_A,w_A))$$
$$+6w_2(Y^{psT}(0,0:0,0|s_A,w_A,n_B))w_2(Y^{pm5}(0,0:0,0:e+1,f|c_B,w_A))+6w_2(Y^{psT}(0,0:0,0|s_A,l_A,n_B))w_2(Y^{pm5}(0,0:0,0:e,f+1|c_B,w_A))$$
$$+4w_3(Y^{psT}(0,0:0,0|s_A,w_A,n_A))w_1(Y^{pm5}(0,0:0,0:e+1,f|c_A,w_A))+4w_3(Y^{psT}(0,0:0,0|s_A,l_A,n_A))w_1(Y^{pm5}(0,0:0,0:e,f+1|c_A,w_A))$$
$$+4w_3(Y^{psT}(0,0:0,0|s_A,w_A,n_B))w_1(Y^{pm5}(0,0:0,0:e+1,f|c_B,w_A))+4w_3(Y^{psT}(0,0:0,0|s_A,l_A,n_B))w_1(Y^{pm5}(0,0:0,0:e,f+1|c_B,w_A))$$
$$+w_4(Y^{psT}(0,0:0,0|s_A,w_A,n_A))P^{sm5}(e+1,f|c_A,w_A)+w_4(Y^{psT}(0,0:0,0|s_A,l_A,n_A))P^{sm5}(e,f+1|c_A,w_A)$$
$$+w_4(Y^{psT}(0,0:0,0|s_A,w_A,n_B))P^{sm5}(e+1,f|c_B,w_A)+w_4(Y^{psT}(0,0:0,0|s_A,l_A,n_B))P^{sm5}(e,f+1|c_B,w_A)$$

Boundary Values:
$w_n(Y^{pm5}(0,0{:}0,0{:}e,f|c_A,w_A))=0$, if (3,0); (3,1); (0,3); (1,3)
$w_n(Y^{pm5}(0,0{:}0,0{:}2,2|c_A,w_A))=w_n(Y^{ps}(0,0{:}0,0|s_A,w_A))$

Table 8 represents the weighted first moment of the number of points remaining in a best-of-5 final advantage set match at set score (e,f) given player A wins the match and is currently serving, where p_A=0.62 and p_B=0.60.

			B score		
		0	1	2	3
	0	165.5	98.7	36.5	0
A score	1	144.4	97.6	43.2	0
	2	90.1	69.8	38.9	
	3	0	0		

Table 8: The weighted first moment of the number of points remaining in a best-of-5 final set advantage match at set score (e,f) given player A wins the match and is currently serving, where p_A=0.62 and p_B=0.60

It follows that:
$w_1(Y^{pm5}(a,b{:}c,d{:}e,f|c_A,w_A))$
$=P^{psT}(a,b{:}c,d|s_A,w_A,n_A)w_1(Y^{pm5}(0,0{:}0,0{:}e+1,f|c_A,w_A))+P^{psT}(a,b{:}c,d|s_A,l_A,n_A)w_1(Y^{pm5}(0,0{:}0,0{:}e,f+1|c_A,w_A))$
$+P^{psT}(a,b{:}c,d|s_A,w_A,n_B)w_1(Y^{pm5}(0,0{:}0,0{:}e+1,f|c_B,w_A))+P^{psT}(a,b{:}c,d|s_A,l_A,n_B)w_1(Y^{pm5}(0,0{:}0,0{:}e,f+1|c_B,w_A))$
$+w_1(Y^{psT}(a,b{:}c,d|s_A,w_A,n_A))P^{sm5}(e+1,f|c_A,w_A)+w_1(Y^{psT}(a,b{:}c,d|s_A,l_A,n_A))P^{sm5}(e,f+1|c_A,w_A)$
$+w_1(Y^{psT}(a,b{:}c,d|s_A,w_A,n_B))P^{sm5}(e+1,f|c_B,w_A)+w_1(Y^{psT}(a,b{:}c,d|s_A,l_A,n_B))P^{sm5}(e,f+1|c_B,w_A)$,
if (c+d) mod 2=0

$w_1(Y^{pm5}(a,b{:}c,d{:}e,f|c_A,w_A))$
$=P^{psT}(a,b{:}c,d|s_B,l_B,n_A)w_1(Y^{pm5}(0,0{:}0,0{:}e+1,f|c_A,w_A))+P^{psT}(a,b{:}c,d|s_B,w_B,n_A)w_1(Y^{pm5}(0,0{:}0,0{:}e,f+1|c_A,w_A))$
$+P^{psT}(a,b{:}c,d|s_B,l_B,n_B)w_1(Y^{pm5}(0,0{:}0,0{:}e+1,f|c_B,w_A))+P^{psT}(a,b{:}c,d|s_B,w_B,n_B)w_1(Y^{pm5}(0,0{:}0,0{:}e,f+1|c_B,w_A))$
$+w_1(Y^{psT}(a,b{:}c,d|s_B,l_B,n_A))P^{sm5}(e+1,f|c_A,w_A)+w_1(Y^{psT}(a,b{:}c,d|s_B,w_B,n_A))P^{sm5}(e,f+1|c_A,w_A)$
$+w_1(Y^{psT}(a,b{:}c,d|s_B,l_B,n_B))P^{sm5}(e+1,f|c_B,w_A)+w_1(Y^{psT}(a,b{:}c,d|s_B,w_B,n_B))P^{sm5}(e,f+1|c_B,w_A)$,
if (c+d) mod 2=1

Similar formulas can be obtained for $w_2(Y^{pm5}(a,b{:}c,d{:}e,f|c_A,w_A))$, $w_3(Y^{pm5}(a,b{:}c,d{:}e,f|c_A,w_A))$ and $w_4(Y^{pm5}(a,b{:}c,d{:}e,f|c_A,w_A))$

Similar derivations can be obtained for $w_n(Y^{pm5}(a,b{:}c,d{:}e,f|c_B,w_A))$, $w_n(Y^{pm5}(a,b{:}c,d{:}e,f|c_A,w_B))$, $w_n(Y^{pm5}(a,b{:}c,d{:}e,f|c_B,w_B))$ and $w_n(Y^{pm5}(a,b{:}c,d{:}e,f|c_B,w_A))$

It follows that $w_n(Y^{pm5}(a,b{:}c,d{:}e,f|c_A))=w_n(Y^{pm5}(a,b{:}c,d{:}e,f|c_A,w_A))+w_n(Y^{pm5}(a,b{:}c,d{:}e,f|c_A,l_A))$

Let $m_n(Y^{pm5}(a,b{:}c,d|c_A)$ represent the n^{th} moment of the number of points remaining in a best-of-5 final set advantage match at point and game score (a,b:c,d) for player A currently serving.

It can be shown that $m_n(Y^{pm5}(a,b{:}c,d|c_A)=w_n(Y^{pm5}(a,b{:}c,d|c_A)$, and thus standard results apply for obtaining parameters of a distribution from moments.

Table 9 represents the mean number of points remaining in a best-of-5 final advantage set match at various score lines for player A serving, where p_A=0.62 and p_B=0.60.

			B score	
		0	1	2
	0	269.4	213.0	124.9
A score	1	197.1	164.8	104.8
	2	106.5	95.3	69.4

Table 9: The mean number of points remaining in a best-of-5 final advantage set match at various score lines for player A serving, where p_A=0.62 and p_B=0.60

Amount of time played in a match

As documented in the ITF 2012 Rules of Tennis "*Between points, a maximum of twenty (20) seconds is allowed. When the players change ends at the end of a game, a maximum of ninety (90) seconds are allowed. However, after the first game of each set and during a tiebreak game, play shall be continuous and the players shall change ends without a rest. At the end of each set there shall be a set break of a maximum of one hundred and twenty (120) seconds*".

To simplify the analysis we will work with a constant rest time such that:
Average Rest Time = Total Rest Time / Number of Points Played

Note that this includes the time between points within a game as well as the time between change of ends at the completion of a game.

To simplify the analysis even further we will assume that the time to play a point is constant for either player serving such that:
Average Time of Point = Average Time of Point (during play) + Average Rest Time

More formally, let $X^{tp}(a,b)$ be a constant random variable of the time to play a point at point score (a,b).

It follows that:
$\mu(X^{tp}(a,b))=X^{tp}(a,b)$
$\sigma^2(X^{tp}(a,b))=0$
$\gamma_1(X^{tp}(a,b))=0$
$\gamma_2(X^{tp}(a,b))=0$

Let $Y^{tm5}(a,b{:}c,d{:}e,f|c_A)$ be a random variable of the amount of time remaining in a best-of-5 final set advantage match at point, game and set score (a,b:c,d:e,f) given player A is currently serving. It follows that:
$\mu(Y^{tm5}(a,b{:}c,d{:}e,f|c_A))=X^{tp}(a,b)\mu(Y^{pm5}(a,b{:}c,d{:}e,f|c_A))$
$\sigma^2(Y^{tm5}(a,b{:}c,d{:}e,f|c_A))=X^{tp}(a,b)^2\sigma^2(Y^{pm5}(a,b{:}c,d{:}e,f|c_A))$
$\gamma_1(Y^{tm5}(a,b{:}c,d{:}e,f|c_A))=\gamma_1(Y^{pm5}(a,b{:}c,d{:}e,f|c_A))$
$\gamma_2(Y^{tm5}(a,b{:}c,d{:}e,f|c_A))=\gamma_2(Y^{pm5}(a,b{:}c,d{:}e,f|c_A))$

Note that the standard results of $\mu(aX) = a\mu(X)$ and $\sigma^2(aX) = a^2\sigma^2(X)$, for a: constant were used in obtaining the above.

Example: It is commonly known that grass is a fast surface with players winning a relatively high percentage of points on serve (and the time to play a point being relatively low in comparison to clay court surfaces). We will proceed to obtain the parameters of distribution for the amount of time remaining in a best-of-5 final set advantage match from the outset for a) grass court match as typically occurs at Wimbledon and b) clay court match as typically occurs at the French Open.

a) p_A=0.72, p_B=0.70, $\mu(X^{tp}(a,b)$=38 secs

b) p_A=0.62, p_B=0.60, $\mu(X^{tp}(a,b)$=47 secs.

These parameters of distribution are given in table 10. Note that the mean amount of time for a clay court match is 33.1 minutes longer on average than for a grass court match. This is somewhat expected due to the increased amount of time to play a point. However the coefficients of skewness and excess kurtosis are greater on grass due to the increased serving dominance which leads to a greater chance of a longer advantage deciding set. Interestingly, the standard deviation is greater on clay, and therefore demonstrates why the standard deviation can be insufficient information for measuring risk.

	Mean (min)	S. Deviation	Variance	Variation	Skewness	Kurtosis
a) Grass	177.9	49.1	2411.7	0.28	1.07	2.55
b) Clay	211.0	49.7	2473.3	0.24	0.25	-0.34

Table 10: The parameters of distribution for the amount of time remaining in a best-of-5 final set advantage match from the outset for a) grass and b) clay

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