Game theoretic solutions to tennis serving strategies

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ABSTRACT

This paper analyzes risk-taking on serve to maximize a player's chances of winning a point on the second serve by either serving a common low risk second serve (with a high second serve percentage) or a high risk second serve by decreasing the second serve percentage but increasing the proportion of points won if the second serve goes in. The notion of "importance" of points is defined and there is evidence to suggest that servers could be encouraged to take more risk on the more "important" points. The results could be used by coaches to help determine how much risk their players should take on the second serve. A working example between Andy Roddick and Rafael Nadal is given to support the results.

Key words: Analysis, risk, serve and return

1. Introduction

Analyzing risk-taking strategies in tennis is complicated. There has been a tendency to analyze risk-taking on the serve more often than other shots. This seems reasonable as the serve is the first shot to be played and therefore simplifies the analysis by not having to consider previous shots in the rally. Barnett et al. (2008) analyzed the situation where players may choose to serve two fast serves by taking into account the type of court surface, and the serving and receiving capabilities of both players. Pollard et al. (2009) extend on this model by allowing for the possibility of players changing serving strategies throughout the match in progress. Consideration of the ideal that a continuum amount of risk is available to players on their serve has further revealed a higher risk first serve and a lower risk second serve strategy as being optimal in most practical situations (Pollard et al., 2007). Pollard (2008) also analyzed the situation in which a medium risk serve (somewhere between a players 'typical' high risk first serve and low risk second serve) has a quadratic rather than linear outcome; one which gives greater weighting to the outcome of serving a high risk serve rather than the outcome of a low risk serve.

All of the above articles analyze the situation where the server is the only decision maker and therefore the optimal strategy will be a single strategy with certainty e.g. a player should always serve a 'typical' high risk first serve on both the first and second serves. When analyzing risk taking on serve by also taking into account whether the receiver is expecting a low or high risk second serve (known more generally as game theory), the optimal strategy can be a mixed strategy e.g. a player should serve a 'typical' high risk first serve 20% of the time on the second serve and a 'typical' low risk second serve 80% of the time on the second serve. This game theory scenario will be analyzed in this article and extended to include the 'importance' of points; where it is suggested for the server to take more risk on the more 'important' points i.e. 30-40 is shown to be the most 'important' point in a game.

2. Data Collection and Analysis

Match statistics from OnCourt (www.oncourt.info) can be obtained for the majority of ATP and WTA matches dating back to 2003. Using a customized program, the average serving and receiving statistics for each player on each surface were calculated, as well as the average head-to-head serving and receiving statistics between any two players.

Bedford et al. (2010) show how a range of statistics (such as the percentage of points won on serve and the 2nd Serve %) can be obtained from the broadcasted match statistics. Table 1 gives the match statistics broadcast from The Artois Championships in 2008 (played on grass) where Rafael Nadal defeated Andy Roddick in two straight sets. Notice that the Serving Points Won is not given directly in the table. This statistic can be derived from the Receiving Points Won such that Serving Points Won for Nadal and Roddick are 1-14/61=77.0% and 1-24/71 =66.2% respectively. Note that the Winning % on 1st Serve is conditional on the 1st Serve going in whereas the Winning % on the 2nd Serve is unconditional on the 2nd Serve going in. The Serving Points Won for Nadal and Roddick along with the Winning % on 1st Serve (uncond.), Winning % on 2nd Serve (cond.) and 2nd Serve % are given in table 2.

	Rafael Nadal	Andy Roddick
1st Serve %	45 of 61 = 73%	46 of 71 = 64%
Aces	7	14
Double Faults	0	3
Winning % on 1st Serve (cond.)	35 of 45 = 77%	34 of 46 = 73%
Winning % on 2nd Serve (uncond.)	12 of 16 = 75%	13 of 25 = 52%
Break Point Conversions	2 of 7 = 28%	0 of 4 = 0%
Receiving Points Won	24 of 71 = 33%	14 of 61 = 22%
Total Points Won	71	61

Table 1: Match statistics between Rafael Nadal and Andy Roddick at The Artois Championships in 2008

	Rafael Nadal	Andy Roddick
Serving Points Won	1-14/61=77.0%	1-24/71 =66.2%
Winning % on 1st Serve (uncond.)	(45/61)*(35/45)=57.4%	(46/71)*(34/46)=47.9%
Winning % on 2nd Serve (cond.)	(12/16)/(1-0/61)=75.0%	(13/25)/(1-3/71)=54.3%
2nd Serve %	1-0/61=100.0%	1-3/71=95.8%

Table 2: Calculated statistics between Rafael Nadal and Andy Roddick at The Artois Championships in 2008

3. Results

Scenario a)

The model developed in Barnett et al. (2008) is used to determine if the server can increase their chances of winning a point by serving high risk on the second serve. As outlined in the introduction this scenario is such that the server is the only decision maker and therefore the optimal strategy will be a single strategy with certainty.

The following definitions are given to obtain a high and low risk serve for each player:

- A high risk serve is a ‘typical’ first serve by a player and calculations are obtained by a player’s averaged percentage of points won on the first serve for a particular surface
- A low risk serve is a ‘typical’ second serve by a player and calculations are obtained by a player’s averaged percentage of points won on the second serve for a particular surface

Note the limitations in these definitions of a high and low risk serve in that to obtain a reasonable sample size a player’s serving statistics is across all players (rather than just head-to-head against the opponent). Also a ‘typical’ first and second serve by each player may not be consistent across each match, but rather a player may be taking more ‘risk’ on the second serve on particular matches for example.

Let:
d_{hijs} = percentage of points won on high risk serves (unconditional) for player i, for when player i meets player j on surface s
d_{lijs} = percentage of points won on low risk serves (unconditional) for player i, for when player i meets player j on surface s

The following two serving strategies are defined:
Strategy 1 – high risk serve followed by a high risk serve
Strategy 2 – high risk serve followed by a low risk serve

Thus, player i should use Strategy 1 (two high risk serves) rather than Strategy 2 if $d_{hijs} > d_{lijs}$

An example of such a case is given in Barnett et al. (2008) between Andy Roddick (recognized as a ‘strong’ server) and Rafael Nadal (recognized as a ‘strong’ receiver), where the results from table 3 indicate that Roddick might be encouraged to serve high risk on both the first and second serve when playing Nadal on grass (since $0.535>0.512$). However he should use a high risk first serve and low risk second serve when playing Nadal on both hard court (since $0.528<0.551$) and clay (since $0.364<0.458$). This example illustrates the fact that it can be important for players to identify the match statistics for themselves and their opponents – specific to court surfaces.

	Andy Roddick			Rafael Nadal		
Statistic	Grass	Hard	Clay	Grass	Hard	Clay
d_{lijs}	0.512	0.551	0.458	0.582	0.571	0.608
d_{hijs}	0.535	0.528	0.364	0.510	0.495	0.546
matches	37	99	17	24	72	72

Table 3: Serving and receiving statistics for Andy Roddick and Rafael Nadal

Scenario b)

The model developed in scenario a) is now extended by taking into account strategies on whether the receiver is expecting a low or high risk second serve. From table 3, where Roddick is serving against Nadal on hard court, Roddick is expected to win 55.1% of points on the second serve when serving low risk on the second serve and expected to win 52.8% of points on the second serve when serving high risk on the second serve. Suppose these percentages are based on whether Nadal on the return of serve is expecting a high or low risk second serve. For example, if Roddick was serving a low risk second serve and Nadal was expecting a low risk second serve, then the percentage won on the second serve for Roddick would likely be less than 55.1%. This is represented in table 4 below in a game theory matrix with the following observation. If Nadal was expecting a low risk second serve 50% of the time and a high risk second serve 50% of the time (indifferent between strategies), then Roddick should always serve a low risk second serve since ½*0.53 + ½*0.57=0.55 and ½*0.55 + ½*0.51=0.53. These results are in agreement with the earlier model from scenario a) where decisions of the opponent were not taken into account.

Using standard game theory techniques to solve this two-person zero-sum game; gives mixed strategies for Roddick of 50% low risk serve, 50% high risk serve and for Nadal of 75% expecting a low risk serve, 25% expecting a high risk serve. The outcome of the game with both players' adopting these mixed strategies is such that Roddick will win 54% of points on the second serve. If either player deviated from these strategies then the other player could capitalize by changing strategies accordingly. For example, if Roddick changed strategies to 80% low risk serve, 20% high risk serve, then Nadal could choose the strategy of 100% expecting low risk serve, for an outcome of Roddick to win 0.53*0.8+0.55*0.2=53.4% of points on the second serve.

		Nadal	
		expecting low risk serve	expecting high risk serve
Roddick	low risk serve	0.53	0.57
	high risk serve	0.55	0.51

Table 4: Game theory matrix of how much risk to take on the second serve in tennis

Scenario c)

The model developed in scenario a) is now extended to include the ‘importance’ of points. The results obtained also extend to the model developed in scenario b). Morris (1977) defines the ‘importance’ of a point for winning a game as the probability that the server wins the game given he wins the next point minus the probability that the server wins the game given he loses the next point. Table 5 gives the ‘importance’ of points to winning the game when the server has a 0.62 probability of winning a point on serve, and shows that 30-40 and Ad-Out are the most ‘important’ points in the game.

	Receiver’s score					
		0	15	30	40	Ad
Server’s score	0	0.25	0.34	0.38	0.28	
	15	0.19	0.31	0.45	0.45	
	30	0.11	0.23	0.45	0.73	
	40	0.04	0.10	0.27	0.45	0.73
	Ad				0.27	

Table 5: ‘Importance’ of points to winning a game when the server has a 0.62 probability of winning a point on serve

The following result follows from Klaassen and Magnus (2001), where it was established that a server’s probability of winning a point decreases with the more ‘important’ points.

Player i should use Strategy 1 (two high risk serves) rather than Strategy 2 if $d_{hijs} > d^\wedge_{lijs}$. The superscript ^ is used as the server’s probability of winning a point on a low risk serve is now conditional on the ‘importance’ of the point.

This is evidence to suggest that the server would be encouraged to take more risk on the more ‘important’ points.

4. Conclusion

The results obtained in this paper could be used by coaches to help determine how much risk their players should take on the second serve. By using the definitions of a high risk serve as a ‘typical’ first serve by each player and a low risk serve as a ‘typical’ second serve by each player, a model where the server was the only decision maker (does not take into account strategies on whether the receiver is expecting a low or high risk second serve) was formulated to determine how much risk a player should take on the second serve. An example was provided between Roddick and Nadal, where it was shown that Roddick might do slightly better when playing Nadal on grass by using two high risk serves rather than using a high risk first serve and a low risk second serve. By establishing a game theory model (by taking into account strategies on whether the receiver is expecting a low or high risk second serve) it was then shown that Roddick against Nadal on hard court could use mixed strategies on serving low and high risk on the second serve, even though the earlier model (that does not take into account strategies on whether the receiver is expecting a low or high risk second serve) indicates that Roddick should be serving low risk on every second serve with certainty for the entire match. Finally, consideration was given to the ‘importance’ of points which then pointed to the server being encouraged to take more risk on the more ‘important’ points.

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