Applying the Kelly criterion to lawsuits

TRISTAN BARNETT†

Faculty of Business and Law, Victoria University, Melbourne, Australia

This article analyses the risks and rewards involved in the litigation process, and whether it is beneficial for a victim to file a lawsuit against the injurer given there are risks involved if unsuccessful in court. The analysis can be used to determine whether a victim should have legal representation in court to obtain a higher expected payout, or minimize risk through legal costs by representing themselves in court, even though the expected payout is reduced without legal representation. Analysis is given to obtain insights as to how much a victim should accept in an out-of-court settlement. A working example from an employment dispute is given to demonstrate the methodology.

Keywords: dispute resolution; litigation; negotiation; casino mathematics; Kelly criterion; employment

1. Introduction

Our daily lives consist of a variety of risk-taking games. They may involve the risk-taking games of blackjack, video poker, sports betting, horse racing or the stock market. There are the less recognized but important risk-taking games of insurance, lawsuits and business. Hence we are often faced with a set of risk-taking games and a reasonable objective across all these games is to increase our current wealth, i.e. grow the size of the bank.

Mathematics is fundamental to solving many problems in industry. Due to the complexity involved in solving industry problems, it can be insightful to break down the problems by finding analogs in games where the mathematics is well-defined. Casino mathematics is one such analog. For example, Barnett and Clarke (2004) found applications to quiz shows by analysing progressive jackpots in video poker machines. Casino mathematics is an example of decision-making under risk where the probabilities can be obtained exactly and the distribution of payouts after a number of trials can be accurately obtained. Therefore, the player is completely aware of the risks involved with the outcomes of the game. When the game is favourable to the player, the famous Kelly criterion formula (Kelly, 1956) can be used to maximize the long-term growth of the bank. In some gambling games, such as sports betting and horse racing, the probabilities are based on estimates, and therefore caution should be taken when applying the Kelly criterion to a favourable game. Despite this caution, the Kelly criterion has been applied successfully to blackjack since Edward Thorp's revolutionary blackjack system (Thorp, 1966), and later to success on the stock market (Thorp and Kassouf, 1967). A detailed account of the story and real-world success behind the Kelly criterion can be found in Poundstone (2005).

---

† Email: strategicgames@hotmail.com

There are many decisions involving risk and uncertainty in industry. One example is whether it is worthwhile to file a lawsuit given there are risks involved from legal fees if unsuccessful in the court trial, and how much to negotiate if an out-of-court settlement is a possibility. To obtain insight to the decision-making process to this problem, a model is developed in this paper that is representative of the structure used in casino games, which utilizes the Kelly criterion.

2. Casino Games

A casino game can be defined as follows: There is an initial cost C to play the game. With the assumption of trials being independent, each trial results in an outcome O_i, where each outcome occurs with profit k_i and probability p_i. The condition $\sum p_i = 1$ must be satisfied. Given this information, the expected profit E_i for each outcome is given by $E_i = p_i k_i$ and the total expected profit is given by $\sum E_i$. The 'percent house margin' (%HM) is then $-\sum E_i / C$ and the total return is $1 + \sum E_i / C$. Positive %HMs indicate that the gambling site on average makes money and the players lose money. Negative %HMs indicate that the game is favourable to the player and could possibly generate a long-term profit. Table 1 summarizes this information when there are m possible outcomes.

2.1 Kelly criterion

The well-established classical Kelly criterion is given by the following result:

Consider a game with two possible outcomes: win or lose, that is played over a 'large' number of trials. Suppose the player profits k units for every unit wager and the probabilities of a win and a loss are given by p and q respectively. Furthermore, suppose that on each trial the win probability p is constant with $p + q = 1$. If $kp - q > 0$, so the game is advantageous to the player, then the optimal fraction of the current capital to be wagered to maximize the long-term growth of the bank is given by $b^* = (kp - q)/k$.

Consider the following example: A player profits \$2 with probability 0.35 and profits −\$1 with probability 0.65, as represented in table 2. Since the total expected profit of $2 \times 0.35 - 0.65 = 0.05 > 0$, the game is advantageous to the player and the optimal fraction is given by $b^* = (2 \times 0.35 - 0.65)/2 = 0.025$. If a player has a \$100 bankroll, then wagering $100 \times 0.025 = \$2.50$ on the next hand will maximize the long-term growth of the bank. If the player loses \$1 on that hand, then under the classical Kelly criterion, the next wager should be exactly $99 \times 0.025 = \$2.475$. Since fractions

TABLE 1 *Representation in terms of expected profit of a casino game with m possible outcomes*

Outcome	Profit	Probability	Expected profit
O_1	k_1	p_1	$E_1 = p_1 k_1$
O_2	k_2	p_2	$E_2 = p_2 k_2$
O_3	k_3	p_3	$E_3 = p_3 k_3$
...	...	...	...
O_m	k_m	p_m	$E_m = p_m k_m$
		1.0	$\sum E_i$

TABLE 2 *A sample casino game to determine the optimal betting fraction under the Kelly criterion*

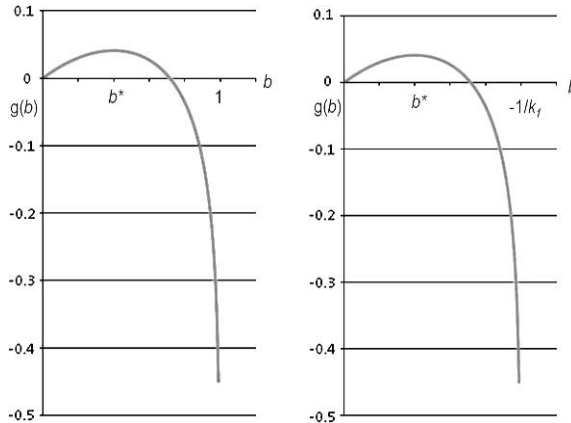| Outcome	Profit	Probability	Expected profit
Win	$2	0.35	$0.70
Lose	−$1	0.65	−$0.65
		1.0	0.05

FIG. 1. Graphical representation of the Kelly criterion for the classical case (left) and when multiple outcomes exist (right), where the optimal betting fraction of b^* occurs at a maximum turning point on $g(b)$.

are often not allowed in gambling games, this figure should be rounded down to an allowable betting amount.

The Kelly criterion when multiple outcomes (more than two) exist is given by the following result:

Consider a game with m possible discrete finite mixed outcomes. Suppose the profit for a unit wager for outcome i is k_i with probability p_i for $1 \leqslant i \leqslant$ m, where at least one outcome is negative and at least one outcome is positive. Then if a winning strategy exists, and the maximum growth of the bank is attained when the proportion of the bank bet at each turn, b, is the smallest positive root of

$$\sum_{i=1}^{m} \frac{k_i p_i}{1 + k_i b} = 0.$$

Let g(b) represent the rate of growth of the bank which is the quantity to be maximized. Figure 1 shows a graphical representation of the Kelly criterion for the classical case (left) and when multiple outcomes exist (right). The player's bank will grow as long as $g(b) > 0$, and is maximized when $g'(b) = 0$ (which is represented by $g(b^*)$ in Fig. 1). It is important to note that a player's bank will not grow (and likely to hit ruin) when overbetting the bankroll, even though the game is still favourable. This is represented on the graph for the values of b such that $g(b) < 0$.

TABLE 3 *The profits and probabilities for the 'All American Poker' game*

Outcome	Return ($)	Profit ($)	Probability	Expected profit ($)
Royal flush	800	799	1 in 43 450	0.018
Straight flush	200	199	1 in 7053	0.028
Four of a kind	40	39	0.00225	0.088
Full house	8	7	0.01098	0.077
Flush	8	7	0.01572	0.110
Straight	8	7	0.01842	0.129
Three of kind	3	2	0.06883	0.138
Two pair	1	0	0.11960	0.000
Jacks or jetter	1	0	0.18326	0.000
Nothing	0	−1	0.58076	−0.581
			1.00	0.0072

2.2 *Video poker*

Video poker is based on the traditional card game of draw poker. Each play of the video poker machine results in five cards being displayed on the screen from the number of cards in the pack used for that particular type of game (usually a standard 52 card pack or 53 if the Joker is included as a wild card). The player decides which of these cards to hold by pressing the hold button beneath the corresponding cards. The cards that are not held are randomly replaced by cards remaining in the pack. The final five cards are paid according to the payout table for that particular type of game. The pay tables follow the same order as traditional draw poker. For example a Full House pays more than a Flush. Without a thorough understanding of video poker, it should be clear in the analysis to follow on how the Kelly criterion with multiple outcomes can be applied to determining an optimal bet size.

A pay table for the outcomes, profits, probabilities and expected profits for a Jacks or Better machine (known as 'All American Poker') are given in table 3. The probabilities were obtained using WinPoker (a commercial product available from the web www.zamzone.com) and assume the player is always maximizing the expected profit on determining the correct playing strategies. Note that $1 is bet each game. It shows that the overall payback for this machine by playing an optimal strategy is 100.72%. The standard deviation is calculated as $5.18. The Kelly criterion is applied to determine a bet size for this video poker game, by using the payouts and probabilities given in table 3. The solver function in Excel is used to calculate this value as $b^* = 0.030679\%$. Example: With a $10,000 bankroll, the Kelly criterion suggests that the player should initially bet $3.07 (likely to be round down to $3).

3. Dispute Resolution

Litigation is a lawsuit filed in a court seeking a legal remedy to the question or dispute existing between the plaintiff and the defendants. The defendants are required to respond to the complaint of the plaintiff. If the plaintiff is successful, judgement will be given in the plaintiff's favour, often resulting in a monetary payout. To avoid the litigation process and hence reduce legal costs between both parties in dispute, a negotiation process may take place to attempt an out-of-court settlement. The other processes in dispute resolution are mediation and arbitration. In mediation, a third party

neutral, known as the mediator, assists the parties in formulating their own resolution of the dispute. Arbitration is an adversarial process whereby an independent third party, after hearing submissions from the disputants, makes an award binding upon the parties.

3.1 *Work agreement*

Disputes can arise from work agreements and requires the victim (employee or contractor in a work context) to recover money from the injurer (business or company). The example used in this paper is based on an actual work agreement that was compiled by the organization and signed by both parties. The following was documented in the work agreement:

Employer: (name of the company)
Employee: (name of the employee)
Terms & Conditions of Employment:

1. *Commencement Date:*
   The date for commencement of duties is Monday 17th July 2006
2. *Remuneration*

   a) *Position Hours*
      The position will be based on 0.6 of an equivalent full time position
   b) *Remuneration*
      On appointment your remuneration will be $500.00 per week which will be paid fortnightly

3. *Probationary Period*
   A probationary period of three months will apply.

The victim was under the impression that he/she was an employee of the company and hence superannuation and holiday pay would apply. When issues were brought up about the type of agreement in 2008, the company stated that he/she was an independent contractor. The company had not issued any tax forms and no tax was taken out. The wages were invariably late, forcing the victim to eventually hand in his/her resignation and be out of the work force. The amount for each query is given in table 4 and shows the total disputed amount of $13000. The situation is complex and many people in the work force would not know the processes involved to best recover the money. Litigation and negotiation processes are now addressed.

TABLE 4 *Type of query with the associated amount for an employment dispute*

Query	Amount ($)
Holiday pay	1000
Late payments	1000
Superannuation	5000
Out of work	6000
Total	13 000

3.2 *Litigation*

The victim is considering filing a lawsuit against the injurer in an attempt to obtain the total disputed amount of $13000. There are risks involved in going to court if unsuccessful. The victim's chances of recovering the money would likely increase with legal representation. However, there are additional legal costs associated with this likely increase in success. It is therefore important to analyse both situations where the victim is representing themselves in court and when a lawyer is acting on the victim's behalf. The total legal cost with legal representation is estimated to be $1800 and by the victim representing themselves in court, the legal cost is $300. Table 5 represents the situation where the victim is represented by a lawyer in court and the structure is in the form of a casino game, as outlined in Section 2. The profits are obtained from the amounts given in table 4. For example, Outcome A was obtained by the total disputed amount less the legal costs ($13\,000 − $1800 = $11\,200) and Outcome B was obtained by adding the amounts for superannuation, late payments and holiday pay less the legal costs ($5000 + $1000 + $1000 − $1800 = $5200). The associated probabilities for each outcome are estimated and in reality could be based on historical data. The game is favourable to the victim with a total expected profit of $3550 (standard deviation of $4078) and could therefore consider filing a lawsuit. However, there are risks involved given a 30% chance of ending up with a loss and a 15% chance of losing $1800. A procedure using the Kelly criterion is now given to assist the victim with the decision as to whether to file a lawsuit.

The Kelly criterion in a gambling context assumes that a player bets a proportion of their current bankroll and overbetting can potentially lead to ruin. Suppose the amount that a player is allowed to bet on each trial is fixed according to the playing rules of the game. The Kelly criterion can still be applied by determining the minimum bankroll requirements such that the player is not overbetting. If A represents the fixed amount to bet on each trial and B represents the player's current bankroll, then a player would not be overbetting in the game only if

$$B \geqslant A/b^* \tag{1}$$

For example, if the 'All American Poker' game from table 3 was fixed at a betting amount of $2.50 for each trial, then a player would not be overbetting in the game if $B \geqslant 2.5/0.030679\% = \$8148.90$.

In the context of litigation (as in the game given in table 5), the amount that the victim is allowed to bet is fixed by the total legal costs and would remain fixed even if the game was played over many trials. The total legal costs are given by the maximum possible loss (MPL) in the representation of the game. Given the well-defined mathematics of the Kelly criterion, the victim's decision as to whether to file a lawsuit could be based on Equation (1). Using Solver in Excel, $b^* = 0.722$. Therefore, the

TABLE 5 *The outcomes of a lawsuit game with legal representation in court*

Outcome	Profit ($)	Probability	Expected profit ($)
A	11200	0.15	1680
B	5200	0.25	1300
C	3200	0.30	960
D	−800	0.15	−120
E	−1800	0.15	−270
		1	3550

victim may consider filing a lawsuit against the injurer if their bankroll is greater than or equal to $1800/0.722 = \$2493$. In general, the victim may consider filing a lawsuit against the injurer if their bankroll is greater than or equal to MPL/b^*. Note that a bankroll is gambling or risk money (that you can afford to lose), as opposed to the money you live on.

Given that the Kelly criterion method is intended for a 'large' number of trials, it should not be too much of a concern that lawsuits usually require only one or two trials. First, the victim (player) may be involved in many favourable risk-taking games such as the stock market, horse racing, blackjack and litigation, which collectively involve many trials. The Kelly criterion can still apply in the litigation context on just the one trial. From an investment perspective, it is important not to hit ruin or be close to losing your bankroll if the worse case happens, i.e. lose the court trial. Therefore, the victim's bankroll needs to be greater than the maximum possible loss. How much greater? That of course can depend on many factors but as a guide or to provide an objective formula, the optimal fraction given by the Kelly criterion is a reasonable estimate since this optimal fraction will be 'considerably' less than one and takes into account the full distribution of the game.

Table 6 represents the situation where the victim is representing themselves in court. The game is favourable to the victim with a total expected profit of \$2600 (standard deviation of \$3390), which is less than the total expected profit of \$3550 given in table 5. However, the legal costs are less than the game given in table 5, where the victim can only lose a maximum of \$300 (compared to \$1800 in table 5). Using Solver in Excel, $b^* = 0.647$. Therefore the victim may consider filing a lawsuit against the injurer if their bankroll is greater than or equal to $300/0.647 = \$464$. It is important to understand the differences in the games given in tables 5 and 6, and often situations arise where obtaining higher expected costs have more associated risks involved.

3.3 *Negotiation*

After the victim has sent a letter of demand or taken legal action by filing a lawsuit with the Magistrates Court, the injurer may want to negotiate an out-of-court settlement. According to the Von Neumann–Morgenstern concept of a utility function (Winston, 1994), there is a lottery such that the victim would be indifferent between a payout of $\$x$ and the game given by the payouts with the associated probabilities as represented in table 5 or table 6. This value of $\$x$ could be interpreted in the legal field as an out-of-court settlement by negotiation.

We will assume that the victim has an adequate bankroll for the game given in table 5 and has taken legal action by sending a letter of demand. The aim is to show that the victim should be willing to accept an amount by negotiation which is less than the total expected profit of \$3550. The Kelly criterion is sensitive to the MPL and the objective is to maximize the long-term growth

TABLE 6 *The outcomes of a lawsuit game with the victim representing themselves in court*

Outcome	Profit ($)	Probability	Expected profit ($)
A	12 700	0.05	635
B	6700	0.1	670
C	4700	0.25	1175
D	700	0.3	210
E	−300	0.3	−90
		1	2600

TABLE 7 *The expected bank of a lawsuit game with legal representation in court*

Outcome	Profit ($)	Probability	Expected bank ($)
A	11 200	0.15	1213
B	5200	0.25	938
C	3200	0.30	693
D	−800	0.15	−87
E	−1800	0.15	−195
		1	2563

of the bank. Minimizing the probability of the MPL can increase the long-term growth of the bank even though the expected profit may be reduced. Maximizing the long-term growth of the bank is equivalent to maximizing the total expected bank. Table 7 gives the expected bank based on the profit outcomes with the associated probabilities in table 5, with $b^* = 0.722$. For example, the expected bank for Outcome A was obtained by $11\,200 \times 0.15 \times 0.722 = \$1213$. Therefore, the total expected bank = total expected profit $\times b^*$. The total expected bank could be used as the minimum amount that the victim should be willing to accept for an out-of-court settlement, and given as $2563 in table 7.

4. Conclusions

This paper has applied the Kelly criterion (as typically used in favourable casino games) to obtain insights in the decision-making process as to whether it is beneficial for a victim to file a lawsuit against the injurer. The analysis can be used to determine whether a victim should have legal representation in court to obtain a higher expected payout, or minimize risk through legal costs by representing themselves in court, even though the expected payout is reduced without legal representation. Analysis was given to obtain insights as to how much a victim should accept in an out-of-court settlement. A working example from an employment dispute was given to demonstrate the methodology.

Employment disputes could also involve alternative dispute resolutions through mediation or arbitration. The Victorian Civil and Administrative Tribunal (VCAT) deals with disputes about the purchase and supply of goods and services. This is an excellent process with minimal fees and the decision is binding upon the parties. However, the work agreement must be written as an independent contractor role for VCAT to consider a tribunal. The Australian Taxation Office (ATO) is another arbitration process where the ATO assess whether the victim was an employee or an independent contractor, and can issue an outstanding money order to the injurer for the superannuation amount. This process is at no cost to the victim, even if the victim is assessed to be an independent contractor by the ATO.

REFERENCES

BARNETT, T. AND CLARKE, S. R. (2004) Optimizing Returns in the Gaming Industry for Players and Operators of Video Poker Machines. Proceedings of the International Conference on Advances in Computer Entertainment Technology. National University of Singapore, 212–216.

KELLY, J. L. (1956) A New Interpretation of Information Rate. The Bell System Technical Journal, July, 917–926.

POUNDSTONE, W. (2005) Fortune's Formula: The Untold Story of the Scientific Betting System That Beat the Casinos and Wall Street. Hill and Wang, New York.

THORP, E. O. (1966) Beat the Dealer: A Winning Strategy for the Game of Twenty One. Random House, New York.

THORP, E. O. AND KASSOUF, S. T. (1967) Beat the Market: A Scientific Stock Market System. Random House, New York.

WINSTON, W. L. (1994) Operations Research: Applications and Algorithms 3rd Edition. International Thompson Publishing, California.