RISK TAKING IN BADMINTON TO OPTIMIZE IN-THE-RUN PERFORMANCE

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Abstract

In tennis the serve can be a most powerful weapon. However in badminton, the serve holds a much lower advantage in comparison to tennis, and for many players, yields a net disadvantage. Badminton's most common service used is a short serve requiring accuracy, as opposed to a long serve requiring power. This is because badminton does not allow for the advantage of a second serve on fault of the first, someway explaining the conservative nature of serving, and low success probabilities. The short serve allows the receiver to gain the advantage, putting the server under pressure on the third shot. In this work, we develop a model to ascertain whether a player should be taking a high or low risk serve. Using Bayesian models, we hypothesize how a player's performance could be optimized conditional on the state of the match in progress. Practical implications for players are discussed, given that the rules of badminton allow for coach intervention during a match in progress.

Keywords: Badminton, risk taking, coach intervention, Markov Chain

1. INTRODUCTION

There are currently three racket sports played at the Olympic Games: tennis and table tennis (which have been played since 1988) and badminton (since 1992). Tennis enjoys world wide popularity, whilst table tennis and badminton are predominately popular in Asia and Europe. Olympic Sports have the highest recognition amongst the vast number of competitive sports that now exist, and as a consequence Badminton Australia are interested in many areas of science (including mathematics) to potentially improve player performance in badminton.

It has been shown in tennis (Barnett et al., 2008) that against certain opponents on specific surfaces, players could possibly increase their chances of winning a point by taking more risk on the second serve. Pollard (2008) considered this problem and found that the range of risk in men's singles tennis serves fall into a quadratic relationship rather than a linear one. Therefore it may not always be optimal to use a 'hard' first serve and a 'soft' second serve. Some players may benefit from taking risky first and second serves, while others may improve results by playing both serves safely. Unlike tennis, only one fault is allowed in badminton, and players generally rely on a low risk short serve, which requires accuracy, rather than a long serve that requires power (Edwards et al., 2005). Badminton players will generally perform better by constantly using a low risk serve, rather than a high risk serve. However, a common strategy amongst players is to occasionally use a high risk serve to catch the opponent off-guard and possibly force them in to making a poor return. This type of strategy has been analysed in tennis using a game theory approach, in which the expectations of the opponent are taken into consideration (Hannan, 1976). In this research we analyse the strategy of when to use an occasional high risk serve throughout a match in progress to potentially enhance player performance. The concept of importance (Morris, 1977) and Bayesian models (Carlin and Louis, 2000) are used in this analysis. Given the rules of badminton allow for coaching intervention during play, this creates great opportunities for live data collection, computer analysis, and intervention which could greatly assist in improving player performance.

2. THE GAME

A badminton match is decided through the best-of-three games. A game can be won only once the score reaches 21 points. If a player reaches 21 and is two points or more ahead, they win the game. If the score reaches 20-20, play continues until one player has obtained a two point lead and is the winner. If the score reaches 29-29, the winner of the next point wins the game. A toss of the coin allows a player to choose the end that they wish to play and whether to serve or receive. The player who wins the point takes serve and thus continues into the following game, with points capable of being won on either a players serve or return of serve. Badminton rules allow for only one service; if a fault is served, the point and service is immediately won by the opponent.

3. MARKOV CHAIN MODEL

Let us consider the possible outcomes of both the game (known as a set in tennis) and winning a match.

3.1 Probability of winning a game

Firstly, to the probability of winning a game. Let p_A and p_B represent the constant probabilities of player A and player B winning a point on serve. Let $S_A(a,b)$ and $S_B(a,b)$ represent the conditional probabilities of player A winning a game, conditional on the point score (a,b), for player A serving and player B serving respectively. These probabilities can be obtained recursively as follows:

$S_A(a,b) = p_A S_A(a+1,b) + (1- p_A)S_B(a,b+1)$
$S_B(a,b) = p_B S_B(a,b+1) + (1- p_B)S_A(a+1,b)$
The boundary values are:

$S_A(a,b) = S_B(a,b) = 1$ if a=21 and b $\leq$19 or (a,b) = (22,20), (23,21), (24,22), (25,23), (26,24), (27,25), (28,26), (29,27), (30,28), (30,29)
$S_A(a,b) = S_B(a,b) = 0$, if b=21 and a $\leq$19 or (a,b) = (20,22), (21,23), (22,24), (23,25), (24,26), (25,27), (26,28), (27,29), (28,30), (29,30)
$S_A(29,29) = p_A$
$S_B(29,29) = 1-p_B$

3.2 Probability of winning a match

Given the match of badminton is the best-of-three games, the outcomes are simple to ascertain. Let m_A and m_B represent the probabilities of player A winning a match when player A and player B are serving first in the match respectively. For notational simplicity, let $S_A(0,0)=s_A$ and $S_B(0,0)=s_B$. Table 1 exhibits the probabilities of player A winning a match for the different game outcomes for when player A and player B are serving first in the match.

Game outcome	Player A serving first	Player B serving first
WW	$s_A s_A$	$s_B s_A$
WLW	$s_A (1-s_A)s_B$	$s_B(1-s_A)s_B$
LWW	$(1-s_A)s_B s_A$	$(1- s_B)s_B s_A$

Table 1: Probabilities of player A winning a match for the different set outcomes

It follows from the table that

$m_A = s_A^2 + 2s_A s_B – 2s_A^2 s_B$
$m_B = s_B^2 + 2s_A s_B – 2s_A s_B^2$

Table 2 exhibits the probabilities of player A winning a game and match for different values of p_A and p_B, and for both player A and player B serving first. It can be observed from the table that when p_A and p_B are greater than 0.5, it is an advantage to serve first in the match. Similarly, when p_A and p_B are less than 0.5, it is an advantage to receive first in the match.

p_A	p_B	s_A	s_B	m_A	m_B
0.46	0.46	0.495	0.505	0.497	0.503
0.48	0.46	0.549	0.556	0.577	0.581
0.50	0.46	0.602	0.607	0.653	0.656
0.52	0.46	0.653	0.655	0.723	0.725
0.48	0.48	0.497	0.503	0.499	0.501
0.50	0.48	0.551	0.554	0.578	0.579
0.52	0.48	0.604	0.604	0.654	0.654
0.50	0.50	0.500	0.500	0.500	0.500
0.52	0.50	0.554	0.551	0.579	0.578
0.52	0.52	0.503	0.497	0.501	0.499

Table 2: Probabilities of player A winning a game and match for different values of p_A and p_B, and for both player A and player B serving first.

3.3 Mean number of points in a game

Let $E_A(a,b)$ and $E_B(a,b)$ represent the mean number of points remaining in a game conditional on the point score (a,b) for player A and player B serving respectively. These probabilities can be obtained recursively as follows:

$$E_A(a,b) = 1+p_A E_A(a+1,b) + (1- p_A) E_B(a,b+1)$$
$$E_B(a,b) = 1+p_B E_B(a,b+1) + (1- p_B) E_A(a+1,b)$$

The boundary values are:
$E_A(a,b) = E_B(a,b) = 0$, if a=21 and b ≤19 or b=21 and a ≤19 or (a,b) = (22,20), (23,21), (24,22), (25,23), (26,24), (27,25), (28,26), (29,27), (30,28), (20,22), (21,23), (22,24), (23,25), (24,26), (25,27), (26,28), (27,29), (28,30)
$E_A(29,29) = E_B(29,29) = 1$

These derivations provide additional interest for both players and coaches during a match in progress.

3.4 Importance of a point to winning a game

Let $I_A(a,b)$ and $I_B(a,b)$ represent the importance of a point to winning a game when player A and player B are serving respectively. $I_A(a,b)$ and $I_B(a,b)$ are defined by Morris (1977) and can be obtained as follows:

$$I_A(a,b) = I_B(a,b) = S_A(a+1,b) – S_B(a,b+1)$$

Note that $I_A(a,b) = I_B(a,b)$ as a result of the rotation of serve in badminton. This result does not occur in a set of tennis since player's alternate serve at the end of each game. It can be shown that $I_A(29,29) = I_B(29,29) = 1$ since the next point decides the winner of the game, and therefore represents the highest level of importance to winning the game.

4. STRATEGIES

We consider a series of strategies to employ whilst a match is in progress.

4.1 Strategy 1 – game theory approach

As stated in the introduction, badminton players will generally perform better by constantly using a low risk serve, rather than a high risk serve. However, a common strategy amongst players is to occasionally use a high risk serve to catch the opponent off-guard and possibly catch them in to making a poor return.

Consider the following between player A (server) and player B (receiver)
a - player A serves low risk and player B is expecting a low risk serve
b - player A serves low risk and player B is expecting a high risk serve
c - player A serves high risk and player B is expecting a low risk serve
d - player A serves high risk and player B is expecting a high risk serve
Based on the above it is reasonable to assign probabilities to the outcomes of a, b, c and d with the condition that d < a < c < b. An example is given in table 3 below in a game theory matrix.

		Player B	
		expected low risk serve	expected high risk serve
Player A	low risk serve	0.55	0.60
	high risk serve	0.57	0.45

Table 3: Game theory matrix of how much risk to take on serve in badminton

Solving this two-person zero-sum game with the Minimax theorem gives mixed strategies for player A of 0.706 low risk serve, 0.294 high risk serve and for player B of 0.882 expecting a low risk serve, 0.118 expecting a high risk serve.

Let p_{iL} represent the proportion of time that player i serves a low risk serve for the match and let p_{iH} represent the proportion of time that player i serves a high risk serve for the match. It follows that $p_{iL} + p_{iH} =1$, given they cover all scenarios. From the example above $p_{iL}=0.706$ and $p_{iH}=0.294$.

According to game theory analysis, this proportion of high risk serves should be randomized. Before the player serves, the coach could signal to the player what type of serve they should use, through a computer based selection.

This form of game theory can also be applied to tennis. For example, even though a tennis player may perform better overall by serving high risk on the first and low risk on the second serve, rather than a high risk on both the first and second, a player may further improve their performance by serving a high risk second serve a proportion of the time as a 'surprise' factor. Game theory analysis could determine the proportion of time that a player should serve out wide, down the line and into the body. Game theory analysis could also determine what proportion of the time that a player should be serve-and-volleying.

4.2 Strategy 2 – risk importance

It is convenient to analyse tennis for devising risk strategies that depend on the level of importance, and then establish the connection to badminton.

It has been established in tennis that the more important the point, the lower the probability that the server wins the point (Klaassen and Magnus, 2001). The model developed in Barnett et al. (2008) and Pollard et al. (2009), is used to determine if the server can increase their chances of winning a point by serving high risk on the second serve.

The following definitions are given for each type of serve:
A high risk serve is a typical first serve by each player
A low risk serve is a typical second serve by each player

Let:
a_{hi} = percentage of high risk serves in play for player i
b_{hi} = percentage of points won on high risk serves (conditional on them being 'in') for player i
b_{li} = percentage of points won on low risk serves (unconditional) for player i
c_{hi} = percentage of points won on return of high risk serves (conditional) for player i
c_{li} = percentage of points won on return of low risk serves (unconditional) for player i
d_{hij} = percentage of points won on high risk serves (conditional) for player i, for when player i meets player j
d_{lij} = percentage of points won on low risk serves (unconditional) for player i, for when player i meets player j
c_{ha}=average percentage (all players) of points won on return of high risk serves (conditional)
c_{la}=average percentage (all players) of points won on return of low risk serves (unconditional)

The following assumptions are given: $a_{hi} < a_{li,}$ $b_{hi} > b_{li}$, $c_{hi} < c_{li}$, $d_{hij} > d_{lij}$, $d_{hij} = b_{hi} - c_{hj} + c_{ha}$ and
$d_{lij} = b_{li} – c_{lj} + c_{la}$

The following two serving strategies are defined:
Strategy 1 – high risk serve followed by a high risk serve
Strategy 2 – high risk serve followed by a low risk serve

The percentage of points won on serve by player i by using each strategy is:
Strategy 1 – $a_{hi} * d_{hij} + (1- a_{hi}) * a_{hi} * d_{hij}$
Strategy 2 – $a_{hi} * d_{hij} + (1- a_{hi}) * d_{lij}$
Thus, player i should use Strategy 1 (two high risk serves) rather than Strategy 2 if
$a_{hi} * d_{hij} + (1- a_{hi}) * a_{hi} * d_{hij} > a_{hi} * d_{hij} + (1- a_{hi}) * d_{lij,}$
and this inequality simplifies to $a_{hi} * d_{hij} > d_{lij}$

The following is developed to determine if the server can increase their chances of winning a point by serving high risk on the second serve, conditional on the "importance" of the point. We will assume that the server's probability of winning a point on serve is affected only by serving low or high risk on the second serve.

The percentage of points won on serve by player i by using a high risk first serve and a low risk second serve is given by:
$a_{hi} * d_{hij} + (1- a_{hi}) * d\hat{}_{lij}$

The superscript ^ is used as the server's probability of winning a point on a low risk serve is now conditional on the "importance" of the point.
From above, $d\hat{}_{lij} = b\hat{}_{li} - c_{lj} + c_{la}$

The following result follows from Klaassen and Magnus (2001), where it was established that a server's probability of winning a point decreases with the more "important points". Given two score lines in tennis x_1 and x_2, if the "importance" at score line x_1 is greater than the "importance" at score line x_2, then $b\hat{}_{li}$ and consequently $d\hat{}_{lij}$ is lesser at score line x_1 than score line x_2.

Player i should use Strategy 1 (two high risk serves) rather than Strategy 2 if $a_{hi} * d_{hij} > d\hat{}_{lij}$. This is evidence to suggest that the server would be encouraged to take more risk on the more "important" points.

Using the above information, we make the following suggestion as a strategy that could be used in men's badminton: *More high risk serves should occur on the more "important" points, and less often on the lesser "important" points.*

Let $p_{iL}(a,b)$ be the proportion of time that player i serves a low risk serve at point score (a,b) and let $p_{iH}(a,b)$ be the proportion of time that player i serves a high risk serve at point score (a,b). The following is discussed based on a game of tennis, to gain insight to obtaining values for $p_{iL}(a,b)$ and $p_{iH}(a,b)$. The average importance of a point to winning a game in tennis (I_{AV}), as defined by Morris (1977), is given as $I_{AV}=N/L$ where $N=dS(0,0)/dp$, L = mean number of points in a game of tennis, $S(0,0)$ represents the probability that the server will win the game at point score (0,0) and p represents the probability that the server wins a point. The following formula is intuitive $p_{iH}(a,b)=[I(a,b)/I_{AV}]p_{iH}$. Note that a game of badminton requires two parameters and therefore a generalized expression for average importance is required.

4.2 Strategy 3 – Bayesian

The proportion of time that a player takes a high and low risk serve can be updated based on the initial proportions and what has occurred during the match using Bayesian analysis. Consider the binomial distribution, with Y the number of events in n independent trials, and μ the event probability. The sampling distribution is defined as $P(Y = y \mid \mu) = {}^{n}C_{y}\, \theta^{y}(1-\theta)^{n-y}$. The posterior distribution of μ given Y is calculated in Carlin and Louis (2000), and is Beta(a,b) with mean $\theta\hat{} = M / (M+n)\ \mu + n / (M+n)\ (Y/n)$, where $M=a+b$, $\mu = a/(a+b)$. Let n_i represent the number of points served by player i, $\mu_i = p_{iH}(a,b)$, Y_i/n_i = the actual percentage of points won on a high risk serve by player i, and M = weighting parameter. Table 4 represents the updated proportion of time that player i should be using a high risk serve for different values of Y_i/n_i and M, given $p_{iH}(a,b) = 0.2$ and $n_i = 5$. Due to the "small" sample size which occurs in the first game of the match, it would appear logical that more weighting should be given towards $p_{iH}(a,b)$ initially, whereas towards the end of the match more weighting should be given to the actual values that occurred during the match. For this reason M=75 appears to be a more reasonable weighting parameter at the start of the match (compared with M=40), whereas M=40 could possibly be a more reasonable weighting parameter towards the end of the match. One method to overcome this problem is to let M_i represent the expected number of serves remaining in the match for player i, where M_i can be obtained from the Markov Chain model in section 3.

	θ^	
Y_i/n_i	M=40	M=75
0	0.18	0.19
1/5	0.20	0.20
2/5	0.22	0.21
3/5	0.24	0.23
4/5	0.27	0.24
1	0.29	0.25

Table 4: The updated proportion of time that player i should be using a high risk serve for different values of Y_i/n_i and M, given $p_{iH}(a,b) = 0.2$ and $n_i = 5$.

5. CONCLUSIONS

In this paper, we outline the simple method of evaluation of success probabilities recursively. From this, game and match success probabilities are evaluated, thereby assisting in the basic merits of differing types of serving strengths. From this, three levels of strategies have been identified in the paper as potential ways to increase player performance.

Strategy 1 is a relatively simple game theory approach to determine when a player should use a high risk serve, with a clear element of surprise approach. Strategy 2 is an extension of Strategy 1. By identifying that more high risk serves should occur on the more "important" points and less often on the lesser "important" points, the adjustment allows for a more specific approach to success. Strategy 3 uses Bayesian analysis to update the initial estimates based on what has occurred during the match, and attempts to optimize in-the-run performance based on current information. All of these strategies can be implemented in a live match, since the rules of badminton allow for coaching intervention during play, and thereby offer technical approaches to success not normally possible in other racquet sports.

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